

CFO Estimation for Subchirp-Mapping Orthogonal Chirp Division Multiplexing in 6G Wireless Communications

Yifei Wang, Yutaka Jitsumatsu

Abstract—With the evolution of sixth-generation (6G) wireless networks, orthogonal chirp division multiplexing (OCDM) has attracted significant research attention due to its robustness against doubly selective channels. However, carrier frequency offset (CFO) impairs the orthogonality of OCDM signals and leads to severe performance degradation, especially under high-mobility scenarios. In conventional full-band OCDM systems, the lack of explicitly reserved structural redundancy limits the exploitation of signal structure. To address this issue, this paper proposes a subchirp-mapping OCDM (SCM-OCDM) framework, in which OCDM-precoded symbols are embedded onto a structured subset of active subchirps, while the remaining subchirps are intentionally left null. By exploiting null subchirps, a null-subchirp-aided (NSA) CFO estimation scheme is developed based on a leakage-energy minimization criterion. The proposed approach effectively exploits the reserved null subchirps, thereby enabling accurate CFO estimation. Simulation results demonstrate that the proposed method achieves accurate CFO estimation and substantial performance gains under high-mobility conditions.

Index Terms—6G, SCM-OCDM, CFO, NSA, leakage-energy minimization, MSE

I. INTRODUCTION

WITH the evolution of 5G-Advanced (5G-A) and sixth-generation (6G) wireless networks, there is a growing demand for waveform designs capable of supporting high data rates, low latency, and reliable communications in emerging applications such as vehicle-to-everything (V2X), intelligent transportation systems, and the Internet of Things (IoT) [1]–[4]. In this context, orthogonal chirp division multiplexing (OCDM) has attracted increasing attention as a potential transmission scheme. By employing orthogonal chirp waveforms instead of sinusoidal subcarriers, OCDM provides improved robustness against doubly selective channels and burst interference compared with conventional orthogonal frequency division multiplexing (OFDM) [5]–[7].

Compared with OFDM, OCDM relies on the inverse discrete Fresnel transform (IDFnT) to spread information symbols across the entire signal block, thereby exhibiting a characteristic double-spreading property [8]. This signal structure provides potential advantages in dispersive wireless environments and improves robustness against carrier impairments. Nevertheless, carrier frequency offset (CFO) remains a critical issue in OCDM systems, since it destroys waveform orthogonality and consequently leads to significant

performance degradation [9]. Therefore, CFO compensation is of great importance for OCDM systems. However, this issue has not been sufficiently investigated in the existing literature. For OFDM systems, CFO estimation has been extensively studied [10]. Although OFDM diagonalizes a frequency-selective channel into parallel subcarriers, channel nulls may still cause multiple CFO candidates to produce similar observations, resulting in estimation ambiguity. To address this issue, reserved null subcarriers have been proposed to improve estimation performance [11]. To facilitate CFO estimation, a frame structure containing null symbols was designed in [12], and a null-symbol-based estimator was developed by exploiting the energy leakage over the null symbols in the Fresnel domain. However, in multipath channels, conventional full-band OCDM spreads the signal energy over all subchirps, making it difficult to preserve null subcarriers for CFO estimation as in OFDM. Therefore, CFO estimation methods relying on null subcarriers become ineffective in full-band OCDM systems. To address this issue, [13] proposed a null-subchirp-based noise-subspace CFO estimation method for OCDM systems by exploiting the covariance matrix of the received signal. However, the structure of the noise subspace depends on both the number of reserved null subcarriers and the channel delay spread. As the channel length is unknown in practice, the noise subspace cannot be accurately determined, which degrades the CFO estimation performance.

To overcome the above limitation, this paper proposes a subchirp-mapping OCDM (SCM-OCDM) transmission scheme for 6G wireless communications. In the proposed framework, subchirp mapping embeds OCDM-precoded symbols into a predefined subset of active subchirps while reserving the remaining subchirps as null subchirps, thereby explicitly introducing a structured null-subchirp configuration. Based on this configuration, a null-subchirp-aided (NSA) CFO estimation method is introduced by exploiting the CFO-induced energy leakage from active subchirps to null subchirps. As a result, the proposed SCM-OCDM scheme achieves reliable CFO estimation without prior knowledge of the channel length, while preserving the essential chirp-domain modulation mechanism of OCDM. Simulation results show that the proposed approach effectively resolves the CFO identifiability issue and yields clear gains in CFO estimation accuracy and bit-error-rate (BER) performance.

Notation: Bold lowercase and uppercase letters denote vectors and matrices, respectively, while calligraphic letters denote sets, e.g., $\mathbf{x}$, $\mathbf{A}$, and $\mathcal{A}$. The notation $[\mathbf{A}]_{m,n}$ denotes the entry in the m -th row and n -th column of matrix $\mathbf{A}$. The identity matrix of size N is denoted by $\mathbf{I}_N$, and the zero matrix of size $M \times N$ is denoted by $\mathbf{0}_{M \times N}$. The operators $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^\dagger$, and $(\cdot)^*$ denote transpose, Hermitian trans-

Yifei Wang is with Department of information science and technology, Graduate School of ISEE, Kyushu University, Japan (e-mail: ywang@me.inf.kyushu-u.ac.jp).

Yutaka Jitsumatsu is with Department of Informatics, Faculty of ISEE, Kyushu University, Japan (jitsumatsu@inf.kyushu-u.ac.jp).

This work was supported in part by JSPS KAKENHI Number JP23K26104 and JP23H00474.

pose, inverse, Moore–Penrose inverse, and complex conjugate, respectively. The expectation operator is denoted by $\mathbb{E}[\cdot]$.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a baseband-equivalent SCM-OCDM system impaired by carrier frequency offset (CFO) over a time-invariant frequency-selective channel.

A. Conventional OCDM System

We consider a conventional OCDM system with a cyclic prefix (CP), consisting of N subchirps. The transmitted symbol vector $\mathbf{x}(i) = [x_1(i), x_2(i), \dots, x_N(i)]^T$ contains both data and pilot symbols in the i -th block. The time-domain signal is generated via IDFT, which is given by

$$\mathbf{s}(i) = \mathbf{\Phi}^H \mathbf{x}(i), \quad (1)$$

where $\mathbf{s} \in \mathbb{C}^{N \times 1}$ represents the time-domain OCDM signal, the operator $(\cdot)^H$ denotes the Hermitian transpose, and $\mathbf{\Phi} \in \mathbb{C}^{N \times N}$ denotes the discrete Fresnel transform (DFnT) matrix, whose (m, n) -th entry is defined as

$$[\mathbf{\Phi}]_{m,n} = \frac{1}{\sqrt{N}} e^{-j\frac{\pi}{4}} \begin{cases} e^{j\frac{\pi}{N}(m-n)^2}, & N = 0 \bmod 2, \\ e^{j\frac{\pi}{N}(m+\frac{1}{2}-n)^2}, & N = 1 \bmod 2, \end{cases} \quad (2)$$

for $m, n = 1, \dots, N$. The matrix $\mathbf{\Phi}$ is circulant and unitary, satisfying $\mathbf{\Phi}^H \mathbf{\Phi} = \mathbf{\Phi} \mathbf{\Phi}^H = \mathbf{I}_N$. Since $\mathbf{\Phi}$ is circulant [5], it can be diagonalized in the Fourier domain as

$$\mathbf{\Phi} = \mathbf{F}^H \mathbf{\Gamma} \mathbf{F}, \quad (3)$$

where $\mathbf{F}$ denotes the normalized discrete Fourier transform (DFT) matrix with entries $[\mathbf{F}]_{k,n} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi kn}{N}}$, $k, n = 1, \dots, N$, and $\mathbf{\Gamma}$ is a diagonal matrix given by

$$\mathbf{\Gamma} = \text{diag} \left\{ e^{-j\frac{\pi}{N}k^2} \right\}_{k=1}^N, \quad (4)$$

By substituting (3) into (1), the conventional OCDM time-domain signal can be rewritten as

$$\mathbf{s}(i) = \mathbf{\Phi}^H \mathbf{x}(i) = \mathbf{F}^H \mathbf{\Gamma}^H \mathbf{F} \mathbf{x}(i). \quad (5)$$

B. Subchirp-Mapping OCDM

As shown in Fig. 2, the SCM-OCDM transmitter first maps the data symbols onto a predefined subset of active subchirps, while the remaining subchirps are reserved as null subchirps. We consider an N -subchirps system with K data subchirps and N_{null} null subchirps,

$$K = N - N_{\text{null}}, \quad (6)$$

Let $\mathbf{x}_d(i) = [x_1(i), x_2(i), \dots, x_K(i)]^T \in \mathbb{C}^{K \times 1}$ denote the K -dimensional data symbol vector in the i -th block. The active-subchirps index set is defined as

$$\mathcal{A}_r = \{a_1, a_2, \dots, a_K\} \subset \{1, 2, \dots, N\}, \quad (7)$$

where the subscript r indicates that the active-subchirps pattern may be random. Accordingly, the null-subchirp index set can be equivalently expressed as the complement of the active-subchirp index set, which is given by

$$\mathcal{G}_r = \{1, 2, \dots, N\} \setminus \mathcal{A}_r = \{g_1, g_2, \dots, g_{N_{\text{null}}}\}, \quad (8)$$

where $\mathcal{A}_r \cap \mathcal{G}_r = \emptyset$ and $\mathcal{A}_r \cup \mathcal{G}_r = \{1, 2, \dots, N\}$. We define the subchirp-mapping matrix $\mathbf{S}_r \in \mathbb{C}^{N \times K}$ as

$$[\mathbf{S}_r]_{n,m} = \begin{cases} 1, & n = a_m, \\ 0, & \text{otherwise}, \end{cases} \quad (9)$$

where $n = 1, 2, \dots, N$ and $m = 1, 2, \dots, K$. By introducing the subchirp-mapping into (4), the transmitted signal of SCM-OCDM in the i -th block can be written as

$$\mathbf{s}(i) = \mathbf{F}_N^H \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i). \quad (10)$$

where $\mathbf{F}_K \in \mathbb{C}^{K \times K}$ is the normalized K -point DFT matrix, $\mathbf{\Gamma}_K \in \mathbb{C}^{K \times K}$ is the quadratic-phase diagonal matrix, and $\mathbf{F}_N^H \in \mathbb{C}^{N \times N}$ is the normalized N -point IDFT matrix.

C. Channel and CFO Model

We consider a time-invariant frequency-selective channel characterized by the channel impulse response (CIR)

$$\mathbf{h} = [h(1), h(2), \dots, h(L)]^T, \quad (11)$$

After CP removal, the received time-domain signal block can be expressed as

$$\mathbf{y}(i) = \mathbf{C}(\epsilon) \mathbf{H} \mathbf{F}_N^H \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i) + \mathbf{w}(i), \quad (12)$$

where $\mathbf{C}(\epsilon) \in \mathbb{C}^{N \times N}$ is the diagonal CFO matrix, $\mathbf{H} \in \mathbb{C}^{N \times N}$ denotes the circulant channel matrix, $\mathbf{w}(i)$ is the additive white Gaussian noise (AWGN) vector, $\mathbf{F}_N^H$ denotes the normalized N -point IDFT matrix, $\mathbf{S}_r$ is the subchirp-mapping matrix.

By applying the N -point DFT to (12), the frequency-domain received signal can be written as

$$\mathbf{Y}(i) = \mathbf{F}_N \mathbf{y}(i) = \mathbf{F}_N \mathbf{C}(\epsilon) \mathbf{H} \mathbf{F}_N^H \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i) + \mathbf{W}(i), \quad (13)$$

where $\mathbf{W}(i) = \mathbf{F}_N \mathbf{w}(i)$. Since $\mathbf{H} \in \mathbb{C}^{N \times N}$ is circulant, it can be diagonalized by the normalized DFT matrix as

$$\mathbf{H} = \mathbf{F}_N^H \mathbf{\Lambda} \mathbf{F}_N, \quad (14)$$

where $\mathbf{\Lambda} \in \mathbb{C}^{N \times N}$ denotes a diagonal matrix, which is given by

$$\mathbf{\Lambda} = \text{diag}(\mathbf{F}_N \mathbf{h}). \quad (15)$$

Therefore, (13) can be equivalently rewritten as

$$\begin{aligned} \mathbf{Y}(i) &= \mathbf{F}_N \mathbf{C}(\epsilon) \mathbf{F}_N^H \mathbf{\Lambda} \mathbf{F}_N \mathbf{F}_N^H \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i) + \mathbf{W}(i) \\ &= \mathbf{F}_N \mathbf{C}(\epsilon) \mathbf{F}_N^H \mathbf{\Lambda} \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i) + \mathbf{W}(i) \\ &= \mathbf{C}_f(\epsilon) \mathbf{\Lambda} \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i) + \mathbf{W}(i), \end{aligned} \quad (16)$$

where $\mathbf{C}_f(\epsilon) = \mathbf{F}_N \mathbf{C}(\epsilon) \mathbf{F}_N^H$.

III. NSA-BASED CFO ESTIMATION

In this section, we develop a null-subchirp-aided (NSA) estimator for the CFO ϵ based on the criterion of minimizing the leakage energy on the null subchirps. For notational convenience, the block index i is omitted in the sequel. Thus, the frequency-domain received signal in (16) can be rewritten as

$$\mathbf{Y} = \mathbf{C}_f(\epsilon) \mathbf{\Lambda} \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d + \mathbf{W}. \quad (17)$$

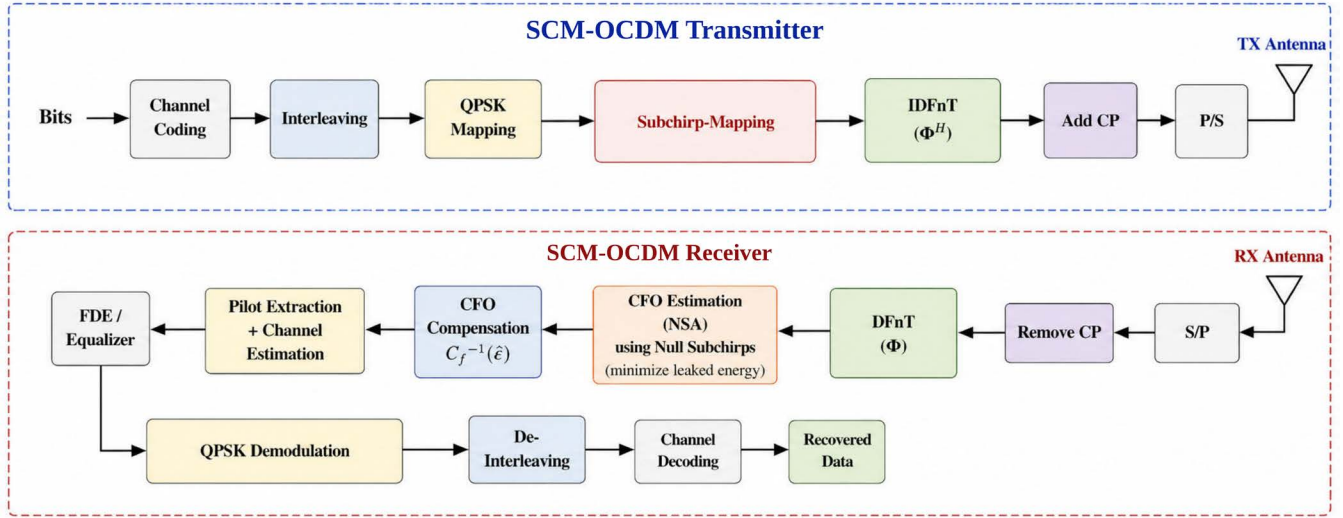


Fig. 1. Transmitter and receiver diagram of block SCM-OCDM system.

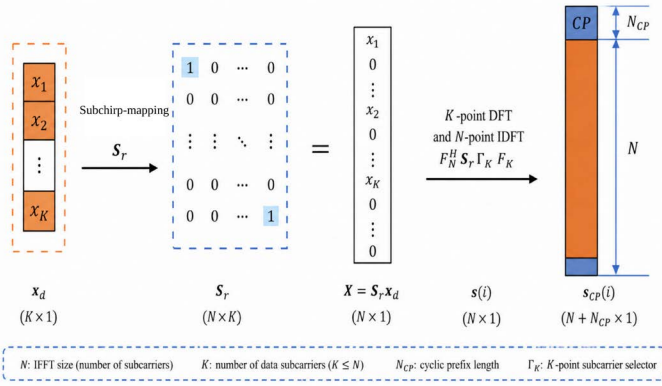


Fig. 2. Subchirp-mapping and time-domain signal generation of SCM-OCDM.

Algorithm 1 Proposed NSA-Based CFO Estimation for SCM-OCDM

Input: Received signal $\mathbf{Y}$, null-subchirp-mapping matrix $\mathbf{S}_0$, candidate CFO search interval $[-\epsilon_{\max}, \epsilon_{\max}]$, and number of search points N_ϵ .

1: Construct the candidate CFO grid $\mathcal{E} = \{\tilde{\epsilon}_1, \tilde{\epsilon}_2, \dots, \tilde{\epsilon}_{N_\epsilon}\}$ uniformly over $[-\epsilon_{\max}, \epsilon_{\max}]$.

for $\ell = 1$ to N_ϵ **do**

2: Form the null-subchirp component after CFO compensation according to (20).

3: Compute the corresponding cost function according to (22).

end for

4: Obtain the CFO estimate by minimizing the cost function according to (22).

Output: Estimated CFO $\hat{\epsilon}$.

where $\mathbf{C}_f(\epsilon)$ denotes the frequency-domain CFO matrix, $\mathbf{\Lambda}$ is the diagonal matrix whose diagonal entries correspond to the frequency-domain channel response.

We define the null-subchirp-mapping matrix $\mathbf{S}_0 \in \mathbb{C}^{N \times N_{\text{null}}}$

as

$$[\mathbf{S}_0]_{n,m} = \begin{cases} 1, & n = g_m, \\ 0, & \text{otherwise,} \end{cases} \quad (18)$$

where $n = 1, 2, \dots, N$ and $m = 1, 2, \dots, N_{\text{null}}$. Then, the received signals observed on the null subchirps are given by

$$\mathbf{Y}_0 = \mathbf{S}_0^T \mathbf{Y}. \quad (19)$$

For a trial CFO value $\tilde{\epsilon}$, the corresponding frequency-domain compensation matrix is written as $\mathbf{C}_f^{-1}(\tilde{\epsilon})$. After CFO compensation, the null-subchirp component becomes

$$\tilde{\mathbf{Y}}_0(\tilde{\epsilon}) = \mathbf{S}_0^T \mathbf{C}_f^{-1}(\tilde{\epsilon}) \mathbf{Y}. \quad (20)$$

Ideally, when $\tilde{\epsilon} = \epsilon$, the CFO-induced leakage from the active subchirps to the null subchirps is minimized. Therefore, the CFO can be estimated by minimizing the residual energy over the null subchirps, which is given by

$$\hat{\epsilon} = \arg \min_{\tilde{\epsilon}} J(\tilde{\epsilon}), \quad (21)$$

where the cost function is defined as

$$J(\tilde{\epsilon}) = \left\| \mathbf{S}_0^T \mathbf{C}_f^{-1}(\tilde{\epsilon}) \mathbf{Y} \right\|_2^2. \quad (22)$$

Since the null-subchirp positions are explicitly predefined by the subchirp-mapping and do not depend on the channel length, the proposed estimator can directly exploit the resulting structured null-subchirp configuration without requiring prior knowledge of the channel length. Once the CFO estimate $\hat{\epsilon}$ is obtained, CFO compensation is first performed on the received signal block. After compensation, the received signal in the frequency domain can be expressed as

$$\hat{\mathbf{Y}}(i) = \mathbf{\Lambda} \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K \mathbf{x}_d(i) + \mathbf{W}(i). \quad (23)$$

For notational simplicity, define

$$\mathbf{B} = \mathbf{\Lambda} \mathbf{S}_r \mathbf{\Gamma}_K \mathbf{F}_K. \quad (24)$$

Then, the data symbols can be recovered by

$$\hat{\mathbf{x}}_d(i) = \mathbf{G} \hat{\mathbf{Y}}(i), \quad (25)$$

TABLE I. Simulation Parameters

System Parameter	Value
System bandwidth	10 MHz
OCDM FFT length N	256, 512, 1024
CP ratio	1/4
Channel model	EVA channel
Modulation method	QPSK
Null Subchirp occupancy ratio ρ	0.1, 0.2, 0.3, 0.4, 0.5

where $\mathbf{G}$ denotes the frequency-domain equalizer. Specifically, the ZF and MMSE equalizers are respectively given by

$$\mathbf{G}_{\text{ZF}} = \mathbf{B}^\dagger, \quad (26)$$

and

$$\mathbf{G}_{\text{MMSE}} = \mathbf{B}^H \left(\frac{\sigma^2}{E_s} \mathbf{I}_N + \mathbf{B}\mathbf{B}^H \right)^{-1}. \quad (27)$$

IV. SIMULATIONS

In this section, simulation results are provided to evaluate the performance of the proposed SCM-OCDM scheme and the associated NSA CFO estimator. The signal-to-noise ratio (SNR) is defined as E_s/σ^2 , where E_s is the average symbol energy and σ^2 is the noise variance. To assess the CFO estimation performance, the mean square error (MSE) is defined as

$$\text{MSE} = \frac{1}{Q} \sum_{q=1}^Q \left(\epsilon^{(q)} - \hat{\epsilon}^{(q)} \right)^2, \quad (28)$$

where Q is the number of Monte Carlo runs, and $\epsilon^{(q)}$ and $\hat{\epsilon}^{(q)}$ denote the true and estimated CFO values in the q -th run, respectively. In each Monte Carlo trial, the normalized CFO ϵ is randomly generated over $[-0.5, 0.5]$, and the candidate CFO grid for the NSA estimator is uniformly sampled over the same interval with step size of $\Delta\epsilon = 0.0005$. At each SNR point, the reported results are averaged over Q independent Monte Carlo runs to ensure statistical reliability. The main simulation parameters are summarized in Table I. The number of Monte Carlo iterations is set to $Q = 100$ in (28). The null-subchirp occupancy ratio is defined as

$$\rho = \frac{N_{\text{null}}}{N}. \quad (29)$$

Fig. 3 illustrates the MSE versus SNR performance of NSA estimation for different null subchirps occupancy ratios ρ , where the DFT length is set at $N = 512$. It can be observed that the estimated MSE decreases monotonically as the SNR increases for all considered values of ρ , which indicates that the proposed estimator benefits significantly from higher SNR conditions. Moreover, a larger occupancy ratio yields better estimation accuracy across the entire SNR range. Specifically, for a target MSE of 10^{-5} , the required SNR is about 15 dB for $\rho = 0.3$, whereas it increases to around 17 dB for $\rho = 0.2$ and 18 dB for $\rho = 0.1$. This corresponds to an SNR gain of approximately 2 dB (compared to $\rho = 0.2$) and 3 dB (compared to $\rho = 0.1$) when increasing the subchirp occupancy ratio. These results indicate that a larger number of null subchirps improves both robustness and estimation accuracy for the proposed NSA scheme.

TABLE II. Power Delay Profile of the EVA Model

Excess tap delay (ns)	0	30	150	310	370	710	1090	1730	2510
Relative Power (dB)	0	-1.5	-1.4	-3.6	-0.6	-9.1	-7.0	-12.0	-16.9

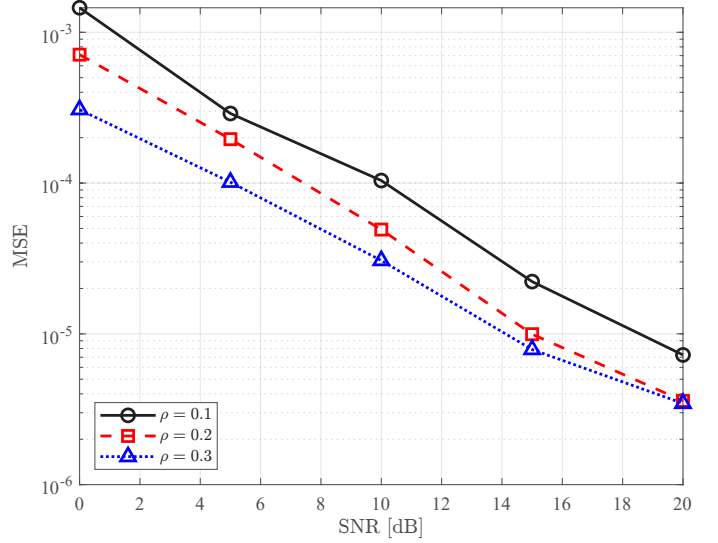


Fig. 3. MSE–SNR results of NSA estimation for different subchirp occupancy ratios ρ .

Fig. 4 illustrates the MSE– ρ performance of NSA estimation for different SNR values, with the DFT length fixed at $N = 512$. It is observed that the estimation MSE decreases monotonically as the subchirp occupancy ratio ρ increases for all considered SNRs. Meanwhile, for any given ρ , a higher SNR yields better estimation accuracy. As ρ increases from 0.1 to 0.3, the MSE is reduced by approximately 5 dB at SNR = 10 dB. However, further increasing ρ beyond 0.3 yields only marginal improvement, particularly in the high-SNR regime where the performance tends to saturate. These results demonstrate that increasing the subchirp occupancy ratio effectively enhances both robustness and estimation accuracy for the proposed NSA estimator, while the gain becomes limited for larger ρ .

Fig. 5 illustrates the BER performance versus SNR for different normalized frequency offsets ϵ , with and without compensation. It can be observed that, for $\epsilon = 0.10$ and $\epsilon = 0.20$, the BER decreases monotonically as the SNR increases in both cases, while the compensated scheme consistently achieves better performance, especially in the medium-to-high SNR region. For example, when $\epsilon = 0.10$, the BER decreases to around 10^{-3} at SNR = 4 dB without compensation, whereas it is further reduced to approximately 2×10^{-4} with compensation. A similar trend can be observed for $\epsilon = 0.20$, where the BER at SNR = 4 dB is about 3×10^{-2} without compensation but drops to around 8×10^{-4} after compensation. In the severe offset case of $\epsilon = 0.45$, the uncompensated BER remains almost unchanged at about 5×10^{-1} over the entire SNR range, indicating a pronounced error floor. By contrast, the proposed compensation effectively suppresses this

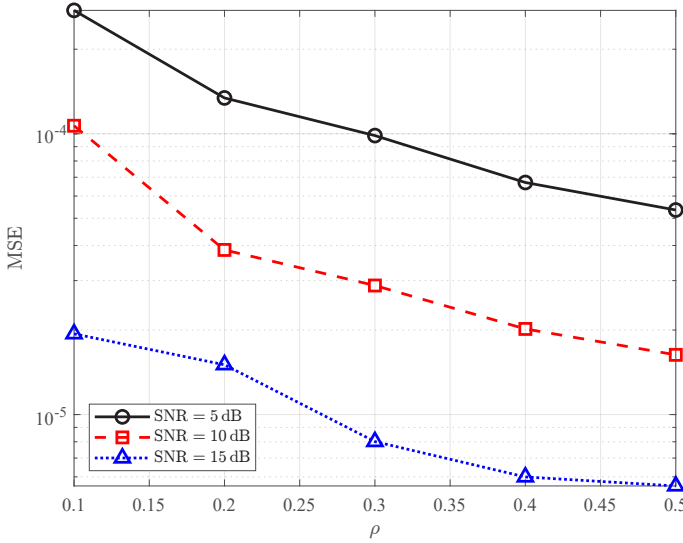


Fig. 4. MSE- ρ results of NSA estimation for different SNR values.

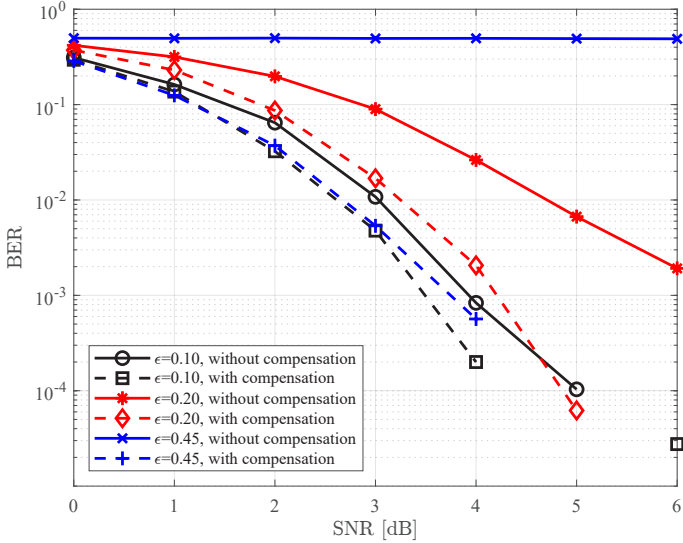


Fig. 5. BER-SNR performance with and without CFO compensation for different CFO values, where $\rho = 0.1$.

degradation and reduces the BER to approximately 2×10^{-4} at SNR = 4 dB. These results demonstrate that the proposed compensation method can substantially mitigate the performance loss caused by frequency offset and significantly improve the reliability of the system, particularly when the offset is large.

V. CONCLUSION

This paper investigated the CFO estimation problem in SCM-OCDM systems. An SCM-OCDM framework was proposed by mapping OCDM-precoded symbols onto a subset of active subchirps while reserving the remaining subchirps as null subchirps. Based on this sparse subchirp structure, a null-subchirp-aided (NSA) CFO estimator was developed using a leakage-energy minimization criterion. The proposed method exploits the reserved null subchirps to identify the

CFO without requiring prior knowledge of the channel length. Simulation results showed that the proposed NSA scheme achieves accurate CFO estimation and significantly improves the BER performance of SCM-OCDM systems under CFO impairment.

REFERENCES

- [1] S. E. Zegrar and H. Arslan, "A novel delay-doppler-based channel estimation method in doubly-dispersive OCDM systems," *IEEE Wireless Communications Letters*, vol. 13, no. 3, pp. 801–804, 2024.
- [2] H. Haif, S. E. Zegrar, and H. Arslan, "Novel OCDM transceiver design for doubly-dispersive channels," *IEEE Transactions on Vehicular Technology*, vol. 73, no. 8, pp. 11 237–11 248, 2024.
- [3] N. Li and P. Fan, "Distributed cell-free massive MIMO versus cellular massive MIMO under ue hardware impairments," *Chinese Journal of Electronics*, vol. 33, no. 5, pp. 1274–1285, 2024.
- [4] X. Ouyang, O. A. Dobre, Y. L. Guan, and J. Zhao, "Chirp spread spectrum toward the nyquist signaling rate—orthogonality condition and applications," *IEEE Signal Processing Letters*, vol. 24, no. 10, pp. 1488–1492, 2017.
- [5] M. S. Omar and X. Ma, "Performance analysis of OCDM for wireless communications," *IEEE Transactions on Wireless Communications*, vol. 20, no. 7, pp. 4032–4043, 2021.
- [6] X. Ouyang and J. Zhao, "Orthogonal chirp division multiplexing," *IEEE Transactions on Communications*, vol. 64, no. 9, pp. 3946–3957, 2016.
- [7] R. Xue and S. Wang, "Combining OMP with a superimposed pilot sparse channel estimation method for underwater acoustic OCDM systems," *IEEE Communications Letters*, vol. 29, no. 9, pp. 2038–2042, 2025.
- [8] D. Lu, Y. Wang, L. Liu, R. Zhang, and X. Ma, "Channel estimation for pilot-aided MIMO-OCDM transmissions," *IEEE Transactions on Communications*, vol. 73, no. 10, pp. 9405–9420, 2025.
- [9] R. Zhang, Y. Wang, and X. Ma, "Channel estimation for OCDM transmissions with carrier frequency offset," *IEEE Wireless Communications Letters*, vol. 11, no. 3, pp. 483–487, 2022.
- [10] X. Ma, C. Tepedelenioglu, G. Giannakis, and S. Barbarossa, "Non-data-aided carrier offset estimators for OFDM with null subcarriers: identifiability, algorithms, and performance," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 12, pp. 2504–2515, 2001.
- [11] M. Ghogho, A. Swami, and G. Giannakis, "Optimized null-subcarrier selection for CFO estimation in OFDM over frequency-selective fading channels," in *GLOBECOM'01. IEEE Global Telecommunications Conference (Cat. No.01CH37270)*, vol. 1, 2001, pp. 202–206 vol.1.
- [12] B. Wang, Y. Wang, Y. Li, and X. Guan, "Underwater acoustic communications based on ocdm for internet of underwater things," *IEEE Internet of Things Journal*, vol. 10, no. 24, pp. 22 128–22 142, 2023.
- [13] S. Guo, Y. Wang, and X. Ma, "Carrier frequency offset estimation for OCDM with null subchirps," *IEEE Signal Processing Letters*, vol. 31, pp. 606–610, 2024.