

Performance Analysis of Multiple Active RIS-Aided Systems With Imperfect Phase Shifts

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Abstract—This work investigates the performance of systems aided by multiple active RISs with imperfect phase shifters over multipath Nakagami- m fading channels, where an active-RIS selection strategy is considered to further enhance the system performance. Under those considerations, the closed expression for outage probability performance is derived. The numerical results not only verify the accuracy of the derived expression but also demonstrate that the proposed active-RIS selection scheme can significantly improve the performance of the considered systems. Therefore, we believe that these observations provide useful insights for future research and practical implementation of active RISs.

Index Terms—Active reconfigurable intelligent surface, outage probability, imperfect phase shifts.

I. INTRODUCTION

To satisfy the stringent requirements of a wide range of Internet of Things (IoT) applications in future wireless communication systems, including remote monitoring, healthcare, autonomous vehicles, and the metaverse [1], reconfigurable intelligent surface (RIS) technology has recently emerged as a promising technology. An RIS typically comprises a large number of elements that can collectively manipulate the phase, amplitude, polarization, or direction of the incident electromagnetic wave through anomalous reflection or refraction theories [2]. Moreover, owing to their compact size, light weight, and low implementation cost, RISs can be flexibly deployed on building facades, walls, vehicles, unmanned aerial vehicles, or aircraft, offering a more energy-efficient and infrastructure-friendly alternative to conventional relay nodes [3].

Nevertheless, conventional passive RIS architectures suffer from a fundamental limitation: the channel degradation over distance. This attenuation is mainly caused by the two-hop propagation mechanism through the RIS, where the transmitted signal first propagates from the source to the RIS and is then redirected from the RIS to the destination [4], [5]. As a result, the effective propagation distance and path loss can be substantially larger than those of the direct link. To mitigate this issue, recent research has increasingly focused on active RIS architectures, in which passive reflecting/refracting elements are enhanced with amplification capabilities [6]. Unlike passive RIS elements, which only adjust the phase

and/or amplitude of the incident wave without introducing signal gain, active RIS elements incorporate amplification circuits together with phase-control components, enabling the redirected signal to be simultaneously phase-adjusted and amplified. Consequently, numerous recent studies have investigated the design, performance analysis, and comparative evaluation of active RIS systems, particularly in relation to their passive RIS counterparts [5]–[8]. These studies show that although active RISs can generally provide better performance than passive RISs, such superiority is not guaranteed under all operating conditions. Deploying an active RIS that fails to outperform its passive counterpart may therefore result in unnecessary hardware complexity, power consumption, and system cost. This is mainly due to two factors: (i) active RISs require more sophisticated fabrication than passive RISs because amplification circuits must be integrated into their reflecting/refracting elements, and (ii) active RISs consume considerably more power than passive RISs since they actively inject additional energy into the redirected signal.

Although active RISs can mitigate the attenuation effect while retaining the main advantages of passive RISs, their integrated amplifiers inevitably introduce additional thermal noise during signal amplification. This creates a fundamental trade-off between passive and active RIS architectures and motivates further research into their appropriate deployment. In this context, this work investigates the best active-RIS selection strategy among multiple deployed active RISs. In practical wireless networks, diverse transceivers and heterogeneous service requirements often make multi-RIS deployment necessary. Typically, only one RIS is selected to assist a given mobile device, whereas the remaining RISs are allocated to other communication links or kept as backups. To capture practical propagation conditions, the transmission channels are modeled using multipath Nakagami- m fading, and imperfect phase shifts at the RIS are considered. The numerical results verify the accuracy of the derived closed-form performance expressions and show that the proposed RIS selection strategy significantly improves system performance compared with arbitrary RIS selection. More importantly, selecting the best active RIS addresses the main limitation of active RISs relative to passive RISs, namely their possible performance loss in high transmit-SNR regimes caused by the additional amplifier noise.

This work was supported by the faculty research fund of Sejong University in 2026.

Notation: $\Gamma(\cdot)$ is the gamma function [9], $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function [9], $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function [9], $\mathcal{U}[\cdot, \cdot]$ is the uniform distribution, and $\mathbb{E}[x]$ and $\mathbb{V}[x]$ present the mean and variance of the variable x , respectively.

II. SYSTEM MODEL

In this work, we consider the multiple active RIS-aided systems, where a single antenna source (S) communicates with a single antenna device (D) with the help of N RIS (R). Each RIS is equipped with K active reflecting elements. For simplicity, we let $\mathcal{T} \in \{S, R\}$ and $\mathcal{R} \in \{R, D\}$. Then, the channel coefficient from $\mathcal{T}$ to $\mathcal{R}$ is given as $h_{\mathcal{TR}}$. Assume that the multi-path Nakagami- m fading models all wireless channels, resulting in the distribution of the channel gain $|h_{\mathcal{TR}}|^2$ is given as

$$f_{|h_{\mathcal{TR}}|^2}(x) = \frac{m_{\mathcal{TR}}^{\mathcal{TR}} x^{m_{\mathcal{TR}}^{\mathcal{TR}}-1}}{\Gamma(m_{\mathcal{TR}}^{\mathcal{TR}}) \Omega_{\mathcal{TR}}^{m_{\mathcal{TR}}^{\mathcal{TR}}}} \exp\left(-\frac{m_{\mathcal{TR}}^{\mathcal{TR}} x}{\Omega_{\mathcal{TR}}}\right), \quad (1)$$

where $m_{\mathcal{TR}}$ and $\Omega_{\mathcal{TR}}$ are the shape and scale parameters, respectively. Then, the received SNR at D with the help of the arbitrary i -th RIS can be represented as

$$\gamma_{\mathcal{D}_i} = \frac{P|\rho \sum_{k=1}^K |h_{\mathcal{SR}}^{k,i}| |h_{\mathcal{RD}}^{k,i}| \phi_e^{k,i}|^2}{\sigma |\rho \sum_{k=1}^K h_{\mathcal{RD}}^{k,i} \phi_e^{k,i}|^2 + 1}, \quad (2)$$

where P is the transmit SNR, $\rho > 1$ is the amplitude gain, $\phi_e^{k,i} = \exp(j\varphi_e^{k,i})$ refers to the imperfect phase-shift process at k -th element of i -th RIS with $\varphi_e^{k,i} \sim \mathcal{U}[-\delta, \delta]$ and $\delta < 2\pi$, σ is the amplifier noise variance to additive white Gaussian noise variance ratio, and $k \in \{1, \dots, K\}$ refers to the k -th element at R. Note that the cases $\sigma = 1$, $\sigma < 1$, and $\sigma > 1$ correspond to scenarios in which the amplifier noise introduced by the RIS elements is equal to, lower than, and higher than the background noise at the receiver, respectively.

Owing to the diversity of devices in a wireless environment, only one active RIS is selected to enhance the signal quality for a given device, whereas the remaining RISs are allocated to other communication links or reserved as backups. Accordingly, the best RIS is selected to serve D, specifically, the RIS that maximizes the received SNR at D. The received SNR from S to D under the best RIS selection strategy can be expressed as

$$\gamma_{\mathcal{D}_n} = \max_{i \in \{1, \dots, N\}} (\gamma_{\mathcal{D}_i}). \quad (3)$$

III. PERFORMANCE ANALYSIS

To evaluate the communication quality of the considered system, this section derives a closed-form expression for the OP performance, which is a widely used metric for assessing the reliability of wireless communication links. Theoretically, OP is defined as the probability that the instantaneous received SNR falls below a predefined threshold, thereby preventing the system from reliably decoding the transmitted data. Mathematically, the OP is defined as

$$\mathcal{O} = \Pr[\gamma_{\mathcal{D}_n} < \gamma_{\text{th}}], \quad (4)$$

where γ_{th} is the targeted threshold.

Theorem 1: A closed-form expression for the OP performance of the considered system is expressed as

$$\mathcal{O} = \left[\frac{\gamma(\kappa, \frac{\gamma_{\text{th}}}{\omega P})}{\Gamma(\kappa)} + \exp\left(\frac{1}{\psi}\right) \frac{\Gamma(\kappa, \frac{\gamma_{\text{th}}}{\omega P})}{\mu^\kappa \omega^\kappa \Gamma(\kappa)} \right]^N, \quad (5)$$

where $\psi = \sigma \rho^2 K \Omega_{\mathcal{RD}}$, $\mu = P/[\gamma_{\text{th}} \psi] + 1/\omega$, $\kappa = \mathbb{E}[H^2]^2 / \mathbb{V}[H^2]$, and $\omega = \mathbb{V}[H^2] / \mathbb{E}[H^2]$ with $H^2 = |\rho \sum_{k=1}^K h_{\mathcal{SR}}^{k,i} h_{\mathcal{RD}}^{k,i} \phi_e^{k,i}|^2$ and $\mathbb{V}[H^2] = \mathbb{E}[H^4] - \mathbb{E}[H^2]^2$. Herein, the detailed derivations for moments of H^2 can be found in [10].

Proof: From the definition in (3) and (4), the OP performance can be calculated as

$$\mathcal{O} = F_{\gamma_{\mathcal{D}_n}}(\gamma_{\text{th}}), \quad (6)$$

where $F_{\gamma_{\mathcal{D}_n}}(t) = (F_{\gamma_{\mathcal{D}_i}}(t))^T$. In other words, the mission now changes to finding out $F_{\gamma_{\mathcal{D}_i}}(t)$, which can be calculated as

$$F_{\gamma_{\mathcal{D}_i}}(t) = \Pr[PH^2/(L^2 + 1) < t], \quad (7)$$

where $L^2 = \sigma |\rho \sum_{k=1}^K h_{\mathcal{RD}}^{k,i} \phi_e^{k,i}|^2$. The probability in (7) can be represented as

$$\begin{aligned} F_{\gamma_{\mathcal{D}_i}}(t) &= \Pr[L^2 > PH^2/t - 1] \\ &= 1 - \Pr[L^2 < PH^2/t - 1] \\ &= 1 - \int_{t/P}^{\infty} F_{L^2}(Px/t - 1) f_{H^2}(x) dx. \end{aligned} \quad (8)$$

According to [7] and [10], the PDF of H^2 and the CDF of L^2 are given, respectively, as

$$f_{H^2}(x) = \frac{x^{\kappa-1}}{\omega^\kappa \Gamma(\omega)} \exp\left(-\frac{x}{\omega}\right), \quad (9)$$

$$F_{L^2}(x) = 1 - \exp\left(-\frac{x}{\psi}\right). \quad (10)$$

Employing $f_{H^2}(x)$ and $F_{L^2}(x)$, $F_{\gamma_{\mathcal{D}_i}}(t)$ can be calculated as

$$\begin{aligned} F_{\gamma_{\mathcal{D}_i}}(t) &= 1 - \int_{t/P}^{\infty} \frac{x^{\kappa-1}}{\omega^\kappa \Gamma(\omega)} \exp\left(-\frac{x}{\omega}\right) dx \\ &\quad + \int_{t/P}^{\infty} \exp\left(-\frac{x}{\omega} - \frac{Px/t}{\psi}\right) \frac{x^{\kappa-1} \exp\left(\frac{1}{\psi}\right)}{\omega^\kappa \Gamma(\omega)} dx. \end{aligned} \quad (11)$$

By using [9, 3.381], $F_{\gamma_{\mathcal{D}_i}}(t)$ can be expressed as

$$F_{\gamma_{\mathcal{D}_i}}(t) = \frac{\gamma(\kappa, \frac{t}{\omega P})}{\Gamma(\kappa)} + \frac{\exp\left(\frac{1}{\psi}\right) \Gamma\left(\kappa, \frac{t[P/(t\psi)+1/\omega]}{P}\right)}{[P/(t\psi)+1/\omega]^\kappa \omega^\kappa \Gamma(\kappa)}. \quad (12)$$

Finally, replacing $F_{\gamma_{\mathcal{D}_i}}(t)$ in (12) with (7) and then using (6), the proof of Theorem 1 is completed. ■

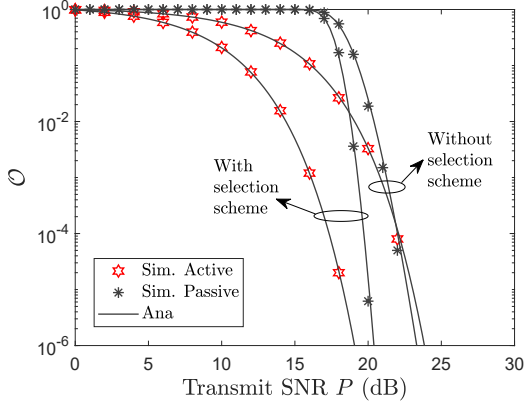


Fig. 1: Effects of N on the OP performance.

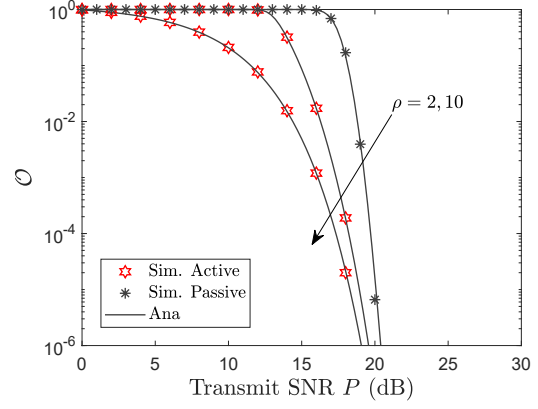


Fig. 2: Effects of ρ on the OP performance.

IV. NUMERICAL RESULTS AND DISCUSSION

This section presents numerical results to validate the derived closed-form expressions. The analytical results are verified through Monte Carlo simulations with 10^6 samples. Unless otherwise specified, the simulation parameters are configured as follows: the number of elements at each active/passive RIS is set to $K = 50$; the number of candidate RISs is $N = 3$; the targeted threshold is $\gamma_{th} = 3$, the amplifier-noise-to-receiver-noise ratio is set to $\sigma = 1$; the Nakagami- m fading parameter is fixed at $m = 4$; the average channel power gain is modeled according to the distance-dependent path-loss model as $\Omega_{\mathcal{TR}} = d_{\mathcal{TR}}^{-\alpha}$, where $d_{\mathcal{TR}}$ denotes the geometric distance between $\mathcal{T}$ and $\mathcal{R}$, and α is the path-loss exponent; the coordinates of $\mathcal{S}$, the RIS, and $\mathcal{D}$ are set to $(0, 0)$, $(4, 5)$, and $(0, 8)$, respectively; and the phase-shift error is assumed to be uniformly distributed over $[-\pi/16, \pi/16]$, corresponding to a maximum phase-shift error of $\delta = \pi/16$.

Fig. 1 illustrates the effect of the RIS selection scheme on the OP performance for the amplitude gain $\rho = 10$. As observed, the RIS selection strategy significantly improves system performance, achieving an approximately 4 dB gain at an OP level of around 10^{-6} . Notably, without RIS selection, active RISs may perform worse than passive RISs in the high transmit-SNR regime due to the additional amplifier noise. However, the drawbacks of active RIS at high transmit SNR P can be mitigated by applying the proposed RIS selection strategy, and even the best active RIS can deliver significantly superior performance compared to its passive counterpart. To justify why passive RISs can outperform active RISs at P in some cases, let us consider the asymptotic high- P expressions. For the active RIS case, the high- P can be expressed as

$$\gamma_D^{\text{Active}} = \frac{P \left| \sum_{k=1}^K |h_{\mathcal{SR}}^{k,i}| |h_{\mathcal{RD}}^{k,i}| |\phi_e^{k,i}|^2 \right|}{\sigma \left| \sum_{k=1}^K h_{\mathcal{RD}}^{k,i} \phi_e^{k,i} \right|^2}, \quad (13)$$

whereas for the passive RIS case, it can be written as

$$\gamma_D^{\text{Passive}} = P \left| \sum_{k=1}^K |h_{\mathcal{SR}}^{k,i}| |h_{\mathcal{RD}}^{k,i}| |\phi_e^{k,i}|^2 \right|. \quad (14)$$

These expressions show that the active-RIS performance is strongly affected by the factor $L^2 = \sigma \left| \sum_{k=1}^K h_{\mathcal{RD}}^{k,i} \phi_e^{k,i} \right|^2$, which captures the impact of the amplifier noise and/or the reflective channels. Specifically, active RISs can outperform passive RISs when $L^2 < 1$, while they may become inferior when $L^2 > 1$. Since L^2 is generally a random quantity that depends on the reflective channel conditions, RIS location, and propagation distances, the active RIS does not always guarantee better performance than the passive RIS. This observation motivates the use of RIS selection, which chooses the active RIS that provides the most favorable end-to-end SNR.

Fig. 2 illustrates the OP performance versus the transmit SNR P under different settings of the amplitude gain ρ . It can be observed that increasing ρ considerably improves the OP performance of the considered systems. Nevertheless, the performance gain becomes less significant as the transmit SNR increases. For example, at OP levels of approximately 10^{-2} , 10^{-4} , and 10^{-6} , increasing ρ reduces the required P by about 3 dB, 1 dB, and 0.5 dB, respectively. This trend is mainly due to the diminishing relative benefit of active RIS amplification at high P levels, where the amplifier-induced noise and normalization effects become more influential.

In summary, Figs. 1 and 2 demonstrate that although active RISs provide substantial signal amplification gains, especially when the amplification factor is large, they may not always outperform passive RISs in the high transmit-SNR regime due to amplifier-induced noise and normalization effects. Nevertheless, the proposed RIS selection strategy enables the system to fully exploit the amplification benefits of active RISs while effectively mitigating their inherent limitations at high transmit-SNR levels.

V. CONCLUSION

This work investigates the performance of multiple active-RIS-aided systems, where a best active-RIS selection strategy is proposed to enhance the received signal quality, rather than relying on random active-RIS selection. The considered system is characterized by multipath Nakagami- m fading

channels, and imperfect phase shifts at the active RIS are incorporated to reflect practical hardware limitations. Based on those considerations, the closed-form expression for the OP is derived. The simulation results confirm the high accuracy of the analytical expression and show that the proposed selection strategy significantly improves the system performance by alleviating the inherent limitations of active RISs, particularly those caused by amplifier-induced noise. These results suggest that the proposed approach offers a promising solution for future active-RIS deployments.

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