

Achievable Rate Maximization for SIM-Assisted SEFDM Systems

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Abstract—With the expected explosive growth of future wireless data traffic, spectrum scarcity has become a critical issue. Spectrally efficient frequency division multiplexing (SEFDM) improves spectral efficiency by reducing subcarrier spacing and enabling spectral overlap. However, the loss of orthogonality causes severe inter-carrier interference, degrading the achievable rate. In this paper, we propose an SEFDM system assisted by a stacked intelligent metasurface (SIM) and investigate a wave-domain phase control method to maximize the achievable rate. In the proposed system, the SIM is deployed at the base station, and the phase shifts of the meta-atoms in each layer are configured to improve the effective channel for SEFDM signals. Simulation results demonstrate that the proposed multilayer SIM structure improves the effective channel of SEFDM signals and achieves a higher achievable rate than a conventional reconfigurable intelligent surface-based system.

Index Terms—SEFDM, stacked intelligent metasurface (SIM), achievable rate maximization, phase shift optimization, wave-domain signal processing

I. INTRODUCTION

With the rapid proliferation of wireless communication devices, global data traffic has been increasing exponentially. To meet this growing demand for communication, wireless communication systems have continually evolved, and fifth-generation mobile communication systems are now being deployed worldwide [1]. One major approach to increasing communication capacity is to exploit higher-frequency bands, such as millimeter-wave and terahertz bands [2], [3]. However, these high-frequency bands suffer from severe propagation loss, posing challenges for coverage extension and practical deployment. In addition to exploiting higher-frequency bands, improving spectral efficiency within the available bandwidth is also important. In this context, spectrally efficient frequency division multiplexing (SEFDM), a non-orthogonal multicarrier transmission scheme, has attracted attention as a promising non-orthogonal extension of orthogonal frequency division multiplexing (OFDM) [4].

SEFDM intentionally relaxes the orthogonality among subcarriers by compressing the subcarrier spacing with a compression factor α . By allocating more subcarriers within the same bandwidth, SEFDM can improve spectral efficiency. However, the spectral overlap among non-orthogonal subcarriers causes inter-carrier interference (ICI), which degrades detection performance and increases receiver processing complexity. To address ICI, extensive studies have been conducted on signal processing techniques at the transmitter and receiver [5]–[9]. *Isam et al.* [8] proposed a precoded SEFDM system based on the discrete

Fourier transform (DFT) and demonstrated the effectiveness of precoding for SEFDM. *Ozan et al.* [11] introduced time-domain precoding for non-OFDM (NOFDM) and showed that ICI can be suppressed even in frequency-selective channels. *Osaki et al.* [9] proposed a method to improve the achievable rate of SEFDM compared with OFDM by using eigenvalue-decomposition-based precoding and power allocation. *Hama et al.* [13] analyzed the achievable spectral efficiency of NOFDM and clarified that the structure of the equivalent signal matrix has a significant impact on spectral efficiency in NOFDM systems. However, such digital precoding approaches require high-dimensional matrix operations, thereby increasing the computational complexity of baseband processing and leading to higher power consumption. Therefore, it is desirable to realize SEFDM-oriented signal processing for ICI mitigation while reducing both computational complexity and power consumption.

To alleviate the interference caused by non-orthogonal signaling without introducing additional RF chains or power-hungry active circuits, reconfigurable intelligent surfaces (RISs) have attracted considerable attention [14], [15]. *Choi et al.* [14] showed that optimizing the RIS reflection phases improves the effective channel condition and enhances the sum rate in faster-than-Nyquist (FTN) signaling. In [15], *Zhang et al.* incorporated RIS into OTFS-FTN transmission for high-mobility scenarios, where RIS phase control was exploited to mitigate FTN-induced interference and enhance spectral efficiency. However, conventional single-layer RISs mainly perform passive beamforming through element-wise reflection coefficient control, which limits their degrees of freedom for wave-domain transformations [16]. To overcome this limitation, stacked intelligent metasurfaces (SIMs), which consist of multiple programmable metasurface layers and exploit inter-layer propagation for signal processing in the electromagnetic (EM) wave domain, have recently attracted attention [17]–[20]. Compared with single-layer RISs, SIMs provide enhanced wave-domain processing capability through cascaded inter-layer propagation and layer-wise phase control. However, the application of SIMs to SEFDM systems remains largely unexplored.

In this paper, we propose a SIM-assisted SEFDM system and establish an achievable-rate maximization framework. Specifically, we formulate the received signal model and derive the achievable rate considering SEFDM-induced ICI and the SIM wave-domain response. We then derive the gradient of the achievable rate and apply a projected gradient ascent

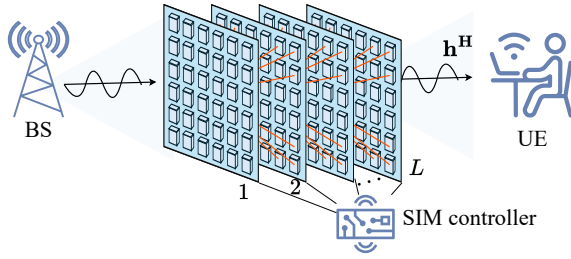


Fig. 1: Transmission model of the stacked intelligent metasurface (SIM).

(PGA) algorithm to address the resulting non-convex problem. Simulation results demonstrate that the proposed SIM-assisted SEFDM system achieves a higher achievable rate than an RIS-based single-layer configuration.

II. SYSTEM MODEL

A. SEFDM Signal Model

The SEFDM symbol carries N data symbols over N subcarriers. Let $\mathbf{s} = [s_0, \dots, s_{N-1}]^T \in \mathbb{C}^{N \times 1}$ denote the data symbol vector, whose entries are independent and identically distributed (i.i.d.). For $k = 0, 1, \dots, N-1$, the k -th sample of the transmitted discrete-time SEFDM signal is given by [21]

$$x[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n \exp\left(\frac{j2\pi n \alpha k}{N}\right), \quad (1)$$

where $x[k]$ denotes the k -th time-domain sample of an SEFDM symbol, and N is the number of subcarriers. The data symbol vector satisfies $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = P/N\mathbf{I}_N$, where P denotes the total transmit power and $\mathbf{I}_N$ denotes the identity matrix of size N . The bandwidth of the SEFDM signal is

$$B_{\text{SEFDM}} = \frac{\alpha(N-1) + 2}{T}, \quad (2)$$

where T denotes the symbol duration, and α is the compression factor satisfying $0 < \alpha \leq 1$. When $\alpha = 1$, the SEFDM signal becomes equivalent to the OFDM signal. Equation (1) can be written in matrix form as

$$\mathbf{x} = \mathbf{\Psi}\mathbf{s}, \quad (3)$$

where $\mathbf{\Psi} \in \mathbb{C}^{N \times N}$ denotes the SEFDM modulation matrix with entries

$$[\mathbf{\Psi}]_{k,n} = \frac{1}{\sqrt{N}} \exp\left(\frac{j2\pi n \alpha k}{N}\right). \quad (4)$$

B. SIM Model

As shown in Fig. 1, we consider a downlink communication system in which an L -layer SIM is deployed at the base station (BS). Each SIM layer consists of $M = x^2$ meta-atoms arranged on a square grid, where x denotes the number of meta-atoms along each side. The EM wave radiated from the BS is incident on the SIM and propagates through the stacked metasurface layers. At the l -th layer, the phase shifts introduced by the M meta-atoms are represented by the following diagonal matrix:

$$\mathbf{\Phi}^l = \text{diag}(e^{j\phi_{l,1}}, e^{j\phi_{l,2}}, \dots, e^{j\phi_{l,M}}), \quad (5)$$

where $\phi_{l,m}$ denotes the phase shift of the m -th meta-atom in the l -th layer.

Let $\mathbf{W}^l(f_i) \in \mathbb{C}^{M \times M}$ denote the propagation matrix from the $(l-1)$ -th layer to the l -th layer at subcarrier frequency f_i . Based on the Rayleigh–Sommerfeld diffraction theory [19], the (m, m') -th entry of $\mathbf{W}^l(f_i)$ is modeled as

$$[\mathbf{W}^l(f_i)]_{m,m'} = \frac{S \cos \chi_{m,m'}^l}{d_{m,m'}^l} \left(\frac{1}{2\pi d_{m,m'}^l} - j \frac{f_i}{c} \right) e^{j2\pi d_{m,m'}^l f_i / c}, \quad (6)$$

where S is the area of each meta-atom, $\chi_{m,m'}^l$ is the incidence angle between the m -th meta-atom in the $(l-1)$ -th layer and the m' -th meta-atom in the l -th layer, $d_{m,m'}^l$ is the corresponding propagation distance, and c is the speed of light. The overall propagation response from the BS to the output of the SIM at subcarrier frequency f_i is then written as

$$\mathbf{T}(f_i) = \mathbf{\Phi}^L \mathbf{W}^L(f_i) \mathbf{\Phi}^{L-1} \dots \mathbf{\Phi}^2 \mathbf{W}^2(f_i) \mathbf{\Phi}^1 \mathbf{W}^1(f_i), \quad (7)$$

where $\mathbf{W}^1(f_i) \in \mathbb{C}^{M \times 1}$ denotes the propagation vector from the BS to the first SIM layer.

C. SIM-Assisted SEFDM Received Signal Model

We model the propagation channel from the SIM to the user equipment (UE) as a quasi-static fading channel. Since spatial correlation can arise among the meta-atoms in the last SIM layer, the channel vector $\mathbf{h} \in \mathbb{C}^{M \times 1}$ is modeled as a correlated Rayleigh fading channel given by [22]

$$\mathbf{h} \sim \mathcal{CN}(0, \mu \mathbf{R}), \quad (8)$$

where μ denotes the path loss between the SIM and the UE, and $\mathbf{R}$ is the normalized spatial correlation matrix among the meta-atoms in the last SIM layer. Under an isotropic scattering assumption, the spatial correlation matrix $\mathbf{R}$ is defined as

$$[\mathbf{R}]_{m,m'} = \text{sinc}\left(\frac{2d_{m,m'}}{\lambda_c}\right), \quad (9)$$

where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$, $d_{m,m'}$ denotes the distance between the m -th and m' -th meta-atoms in the same layer, and $\lambda_c = c/f_c$ is the wavelength at the carrier frequency f_c .

Using (7) and (8), the SIM-assisted subcarrier response from the BS to the UE at frequency f_i is expressed as

$$H_{\text{SIM}}(f_i) = \mathbf{h}^H \mathbf{\Phi}^L \mathbf{W}^L(f_i) \dots \mathbf{\Phi}^2 \mathbf{W}^2(f_i) \mathbf{\Phi}^1 \mathbf{W}^1(f_i). \quad (10)$$

Let $\mathbf{H}_{\text{SIM},D} \triangleq \text{diag}(H_{\text{SIM}}(f_0), H_{\text{SIM}}(f_1), \dots, H_{\text{SIM}}(f_{N-1})) \in \mathbb{C}^{N \times N}$ denote the SIM-assisted frequency-domain channel matrix sampled on the DFT grid. Then, the received time-domain signal $\mathbf{y} \in \mathbb{C}^{N \times 1}$ can be written as [23]

$$\mathbf{y} = \mathbf{D}^H \mathbf{H}_{\text{SIM},D} \mathbf{D} \mathbf{x} + \mathbf{n}, \quad (11)$$

where $\mathbf{D}$ is the unitary DFT matrix, and $\mathbf{n}$ is additive white Gaussian noise (AWGN) with zero mean and covariance $\mathbb{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_N$.

At the UE, the received SEFDM signal is demodulated by a matched filter (MF). The MF output $\mathbf{r} \in \mathbb{C}^{N \times 1}$ is given by

$$\mathbf{r} = \mathbf{\Psi}^H \mathbf{D}^H \mathbf{H}_{\text{SIM},D} \mathbf{D} \mathbf{x} + \tilde{\mathbf{n}} = \mathbf{G} \mathbf{s} + \tilde{\mathbf{n}}, \quad (12)$$

where $\tilde{\mathbf{n}} = \mathbf{\Psi}^H \mathbf{n}$ is zero-mean noise with covariance $\mathbb{E}[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H] = \sigma^2 \mathbf{C}$, $\mathbf{G} = \mathbf{\Psi}^H \mathbf{D}^H \mathbf{H}_{\text{SIM},D} \mathbf{D} \mathbf{\Psi}$, and $\mathbf{C} = \mathbf{\Psi}^H \mathbf{\Psi}$ is the SEFDM correlation matrix. The (m, n) -th element of $\mathbf{C}$ is given by

$$[\mathbf{C}]_{m,n} = \begin{cases} 1, & m = n, \\ \frac{1}{N} \frac{1 - \exp(j2\pi\alpha(m-n))}{1 - \exp(j\frac{2\pi\alpha(m-n)}{N})}, & m \neq n. \end{cases} \quad (13)$$

The diagonal elements of $\mathbf{C}$ are equal to one, whereas the off-diagonal elements correspond to ICI components and are generally nonzero when $\alpha < 1$. After whitening, the MF output becomes

$$\bar{\mathbf{r}} = \mathbf{C}^{-\frac{1}{2}} \mathbf{G} \mathbf{s} + \tilde{\mathbf{n}} = \tilde{\mathbf{G}} \mathbf{s} + \tilde{\mathbf{n}}, \quad (14)$$

where $\tilde{\mathbf{n}}$ is zero-mean white noise with covariance $\mathbb{E}[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H] = \sigma^2 \mathbf{I}_N$, and $\tilde{\mathbf{G}} = \mathbf{C}^{-\frac{1}{2}} \mathbf{G}$ denotes the whitened effective channel. From (14), the achievable rate of the SIM-assisted SEFDM system is expressed as

$$R = \frac{1}{\alpha N} \log_2 \det \left(\mathbf{I}_N + \frac{P}{\sigma^2 N} \tilde{\mathbf{G}} \tilde{\mathbf{G}}^H \right). \quad (15)$$

Equation (15) shows that the achievable rate is characterized by the effective channel $\tilde{\mathbf{G}}$, which includes ICI components among subcarriers. Therefore, in this paper, we control the SIM phase shift matrices $\{\mathbf{\Phi}^l\}_{l=1}^L$ to realize EM-domain analog precoding, thereby shaping the effective channel and maximizing the achievable rate of SEFDM signals.

III. PROBLEM FORMULATION AND PROPOSED SOLUTION FOR THE SIM-ASSISTED SEFDM SYSTEM

A. Problem Formulation

In this paper, we optimize the phase shifts of the SIM deployed at the BS to maximize the achievable rate at the UE. To focus on the phase optimization performance, perfect channel state information (CSI) is assumed to be available. Let $\phi^l \triangleq [\phi_{l,1}, \phi_{l,2}, \dots, \phi_{l,M}]^T \in \mathbb{R}^{M \times 1}$ denote the phase shift vector of the l -th SIM layer, and let $\varphi \triangleq \{\phi^1, \phi^2, \dots, \phi^L\}$ denote the collection of phase shift vectors across all SIM layers. Then, the EM-domain phase optimization problem is formulated as

$$(\mathcal{P}) : \max_{\varphi} R = \frac{1}{\alpha N} \log_2 \det \left(\mathbf{I}_N + \frac{P}{\sigma^2 N} \tilde{\mathbf{G}}(\varphi) \tilde{\mathbf{G}}^H(\varphi) \right), \quad (16a)$$

$$\text{s.t. } \phi_{l,m} \in [0, 2\pi), \quad (16b)$$

Problem $(\mathcal{P})$ is generally non-convex because the achievable rate is a nonlinear function of the SIM phase shifts. In the next subsection, we develop an iterative phase optimization algorithm based on projected gradient ascent (PGA) to obtain a suboptimal solution for the SIM-assisted SEFDM system.

Algorithm 1 Projected Gradient Ascent Algorithm for SIM Phase Optimization

- 1: **Input:** $\{\mathbf{W}^l(f_i)\}_{i=1}^L, \mathbf{h}$.
- 2: Initialize φ that satisfies $\phi_{l,m} \in [0, 2\pi)$.
- 3: Compute the initial achievable rate R in (15).
- 4: **Repeat**
- 5: Compute the gradient $\partial R / \partial \phi_{l,m}$ by using (20).
- 6: Determine the step size β by backtracking line search.
- 7: Update φ using (21).
- 8: **Until** The achievable rate R in (15) converges.
- 9: **Output:** Optimized phase shifts φ and achievable rate R .

B. Projected Gradient Ascent Algorithm for SIM Phase Optimization

For the constrained non-convex problem $(\mathcal{P})$, we employ a PGA algorithm that iteratively updates the SIM phase shifts. At each iteration, the achievable-rate gradient with respect to all phase shifts $\phi_{l,m}$ is evaluated, and all phases are simultaneously updated by a projected ascent step. The procedure of the PGA algorithm is summarized in Algorithm 1. Let $\varphi^{(t)}$ denote the phase variables at the t -th iteration. Given the current phase shifts, the effective channel $\tilde{\mathbf{G}}(\varphi^{(t)})$ is first computed, and the achievable rate $R^{(t)}$ is obtained from (15). Then, the gradient of the achievable rate with respect to each phase shift $\phi_{l,m}$ is evaluated.

For the gradient derivation, we decompose the cascaded SIM response at the l -th layer. For subcarrier frequency f_i , let $\mathbf{u}_{i,l}^H$ denote the cascaded response after the l -th phase shift matrix toward the UE, and let $\mathbf{v}_{i,l}$ denote the cascaded response before the l -th phase shift matrix from the BS side. These vectors are defined as

$$\mathbf{u}_{i,l}^H \triangleq \mathbf{h}^H \mathbf{\Phi}^L \mathbf{W}^L(f_i) \dots \mathbf{\Phi}^{l+1} \mathbf{W}^{l+1}(f_i) \in \mathbb{C}^{1 \times M}, \quad (17)$$

$$\mathbf{v}_{i,l} \triangleq \mathbf{W}^l(f_i) \mathbf{\Phi}^{l-1} \mathbf{W}^{l-1}(f_i) \dots \mathbf{\Phi}^1 \mathbf{W}^1(f_i) \in \mathbb{C}^{M \times 1}. \quad (18)$$

For the boundary layers, we set $\mathbf{u}_{i,L}^H = \mathbf{h}^H$ and $\mathbf{v}_{i,1} = \mathbf{W}^1(f_i)$. The components associated with the m th meta-atom in the l th layer are denoted by $u_{i,l,m} = [\mathbf{u}_{i,l}^H]_m$ and $v_{i,l,m} = [\mathbf{v}_{i,l}]_m$, respectively.

The derivative of the SIM-assisted frequency-domain channel matrix with respect to $\phi_{l,m}$ is given by

$$\begin{aligned} \mathbf{\Lambda} &= \frac{\partial \mathbf{H}_{\text{SIM},D}}{\partial \phi_{l,m}} \\ &= \text{diag} \left(j e^{j\phi_{l,m}} u_{0,l,m} v_{0,l,m}, \dots, j e^{j\phi_{l,m}} u_{N-1,l,m} v_{N-1,l,m} \right). \end{aligned} \quad (19)$$

Using $\mathbf{\Lambda}$, the gradient of the achievable rate with respect to $\phi_{l,m}$ is obtained as

$$\frac{\partial R}{\partial \phi_{l,m}} = \frac{2\rho}{\alpha N \ln 2} \text{Re} \left\{ \text{tr} \left(\tilde{\mathbf{G}}^H \mathbf{\Lambda}^{-1} \mathbf{C}^{-\frac{1}{2}} \mathbf{\Psi}^H \mathbf{D}^H \mathbf{\Lambda} \mathbf{D} \mathbf{\Psi} \right) \right\}, \quad (20)$$

where $\rho = \frac{P}{\sigma^2 N}$ and $\mathbf{A} = \mathbf{I}_N + \rho \tilde{\mathbf{G}} \tilde{\mathbf{G}}^H$. After computing the gradients for all meta-atoms using (20), the phase shifts are updated simultaneously as

$$\phi_{l,m} \leftarrow \mathcal{P}_{\mathcal{F}} \left(\phi_{l,m} + \beta \frac{\partial R}{\partial \phi_{l,m}} \right), \quad (21)$$

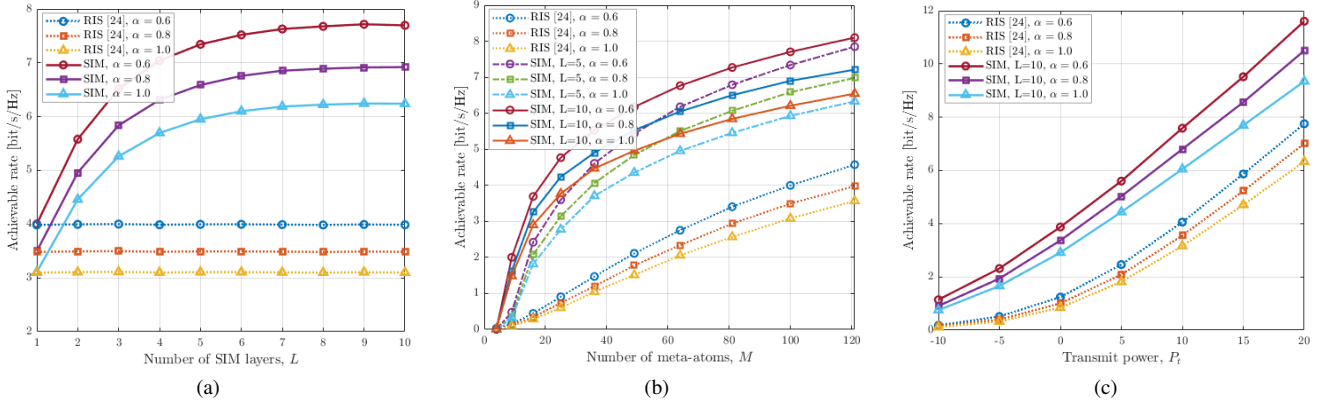


Fig. 2: Achievable rate comparison between the proposed SIM and the RIS [25]: (a) Achievable rate versus the number of SIM layers L ($P_t = 10$ dBm), (b) Achievable rate versus the number of meta-atoms M ($P_t = 10$ dBm), (c) Achievable rate versus the transmit power P_t ($M = 100$).

where $\beta > 0$ is the step size determined by Armijo backtracking line search [24]. The operator $\mathcal{P}_{\mathcal{F}}(\cdot)$ wraps the updated phase into the feasible interval $\mathcal{F} = [0, 2\pi)$. The above update is repeated until the relative increase in the achievable rate falls below a prescribed threshold.

IV. SIMULATION RESULTS

In this section, we evaluate the achievable rate of the proposed SIM phase optimization method by varying the number of SIM layers L , the number of meta-atoms M , the transmit power P_t , and the SEFDM bandwidth compression factor α . For comparison, we consider an RIS-assisted configuration represented by a single-layer phase control surface [25]. In this paper, this benchmark is implemented as the $L = 1$ case of the considered SIM structure. This comparison is used to assess the effect of multilayer wave-domain transformations on the achievable rate.

In the simulations, the carrier frequency is set to $f_c = 28$ GHz, and the corresponding wavelength is $\lambda_c = c/f_c$. The number of SEFDM subcarriers is $N = 16$, and the system bandwidth is $B = 10$ MHz. The bandwidth compression factor is varied over $\alpha \in \{0.6, 0.8, 1.0\}$. The SIM has L layers, each containing $M = x^2$ meta-atoms arranged on a square grid. The spacing between adjacent meta-atoms is set to $\lambda_c/2$, and the area of each meta-atom is $(\lambda_c/2)^2$. The total thickness of the SIM is set to $D_{\text{SIM}} = 10\lambda_c$, and the inter-layer spacing is given by D_{SIM}/L . The SIM-UE channel follows the correlated Rayleigh fading model in (8). The noise power is $\sigma^2 = -104$ dBm, and the BS transmit antenna gain is 5 dBi. The SIM phase shifts are optimized using the PGA algorithm described in Section III.

A. Achievable Rate Performance versus the Number of SIM Layers

Fig. 2(a) shows the achievable rate performance of the multilayer SIM and RIS configurations versus the number of SIM layers L . For the RIS configuration, the total number of meta-atoms in the corresponding SIM, i.e., ML , is placed on

a single metasurface layer. To maintain the same aperture size, the meta-atom spacings of the RIS and SIM configurations are set to $5\lambda_c/\sqrt{ML}$ and $5\lambda_c/\sqrt{M}$, respectively, where the number of meta-atoms per SIM layer is fixed at $M = 100$.

As shown in Fig. 2(a), increasing the total number of meta-atoms in the RIS configuration provides only a limited improvement in the achievable rate. This is mainly due to the increased spatial correlation caused by the reduced meta-atom spacing within the fixed aperture. In contrast, the multilayer SIM configuration improves the achievable rate as L increases. In particular, when $L = 7$, the multilayer SIM achieves approximately twice the achievable rate of the corresponding RIS configuration. The cascaded multilayer structure of the SIM enables high-dimensional wave-domain transformations that are difficult to realize with a single-layer phase control surface.

B. Achievable Rate Performance versus the Number of Meta-atoms

Fig. 2(b) shows the achievable rate performance versus the number of meta-atoms M . In this evaluation, the spacing between adjacent meta-atoms in each metasurface layer is fixed at $d = \lambda_c/2$, while M is varied.

For all configurations, the achievable rate increases with M . This trend indicates that increasing the number of meta-atoms enhances the spatial degrees of freedom for phase control, allowing the SIM to more flexibly manipulate the propagating EM waves. The SIM configurations with $L = 5$ and $L = 10$ achieve higher achievable rates than the RIS configuration. Even for the same M , the multilayer SIM provides a larger rate gain than the RIS, indicating that the layer-domain degrees of freedom also contribute to improving the achievable rate. However, the rate improvement gradually becomes smaller as M increases. Furthermore, for all values of M and L , a smaller bandwidth compression factor α results in a higher achievable rate.

C. Achievable Rate Performance versus Transmit Power

Fig. 2(c) shows the achievable rate performance versus the transmit power P_t . In this evaluation, the RIS configuration is compared with the multilayer SIM configuration with $L = 10$. As in Fig. 2(a), the RIS configuration is implemented by placing the same total number of meta-atoms as the multilayer SIM on a single metasurface layer, with the meta-atom spacing adjusted to maintain a comparable aperture size.

As shown in Fig. 2(c), the achievable rate monotonically increases with P_t for all configurations. This behavior follows from (15), where increasing $P/(\sigma^2 N)$ improves the received signal-to-noise ratio. Moreover, the multilayer SIM with $L = 10$ achieves a higher achievable rate than the RIS configuration over the entire transmit power range. This result is consistent with Fig. 2(a), indicating that distributing meta-atoms across multiple layers is more effective for SEFDM transmission than densely placing the same total number of meta-atoms on a single layer. A smaller bandwidth compression factor α further improves the achievable rate, and the multilayer SIM with $\alpha = 0.6$ achieves the highest rate among the considered cases.

V. CONCLUSION

In this paper, we proposed an achievable rate maximization method for SIM-assisted SEFDM systems. The received signal model of SEFDM signals passing through the SIM was formulated, and a projected gradient ascent-based algorithm was developed to optimize the SIM phase shifts. Simulation results showed that the proposed multilayer SIM structure achieves a higher achievable rate than the RIS configuration. In particular, the results indicated that distributing meta-atoms across multiple layers can improve the achievable rate of SEFDM signals by exploiting both inter-layer propagation and layer-wise phase control. These results demonstrate that wave-domain signal processing using multilayer SIMs is also effective for improving the achievable rate of SEFDM systems.

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REFERENCES

- [1] J. Qiao, X. S. Shen, J. W. Mark, Q. Shen, Y. He, and L. Lei, "Enabling device-to-device communications in millimeter-wave 5G cellular networks," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 209–215, Jan. 2015.
- [2] M. Inomata, W. Yamada, N. Kuno, M. Sasaki, K. Kitao, M. Nakamura, H. Ishikawa, and Y. Oda, "Terahertz propagation characteristics for 6G mobile communication systems," in *Proc. Eur. Conf. Antennas Propag. (EuCAP)*, 2021, pp. 1–5.
- [3] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101–107, Jun. 2011.
- [4] H. Tataria, M. Shafi, A. F. Molisch, M. Dohler, H. Sjöland, and F. Tufvesson, "6G wireless systems: Vision, requirements, challenges, insights, and opportunities," *Proc. IEEE*, vol. 109, no. 7, pp. 1166–1199, Jul. 2021.
- [5] I. Darwazeh, H. Ghannam, and T. Xu, "The first 15 years of SEFDM: A brief survey," in *Proc. 11th Int. Symp. Commun. Syst., Netw. Digit. Signal Process. (CSNDSP)*, 2018, pp. 1–7.
- [6] M. Rodrigues and I. Darwazeh, "A spectrally efficient frequency division multiplexing based communications system," in *Proc. 8th Int. OFDM-Workshop (InOWo'03)*, Hamburg, Germany, 2003, pp. 48–49.
- [7] S. Isam and I. Darwazeh, "Characterizing the intercarrier interference of non-orthogonal spectrally efficient FDM system," in *Proc. 8th Int. Symp. Commun. Syst., Netw. Digit. Signal Process. (CSNDSP)*, Poznan, Poland, 2012, pp. 1–5.
- [8] S. Isam and I. Darwazeh, "Precoded spectrally efficient FDM system," in *Proc. IEEE 21st Int. Symp. Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Istanbul, Turkey, Sep. 2010, pp. 99–104.
- [9] S. Osaki, T. Ishihara, and S. Sugiura, "Eigenvalue-decomposition-precoded ultra-dense non-orthogonal frequency-division multiplexing," *IEEE Trans. Wireless Commun.*, vol. 19, no. 12, pp. 8165–8178, Dec. 2020.
- [10] H. Ghannam and I. Darwazeh, "Design and performance of SEFDM signals with power allocation," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Marrakesh, Morocco, Apr. 2019, pp. 1–6.
- [11] W. Ozan, P. A. Haigh, B. Tan, and I. Darwazeh, "Time precoding enabled non-orthogonal frequency division multiplexing," in *Proc. IEEE 30th Annu. Int. Symp. Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Istanbul, Turkey, Sep. 2019, pp. 1–6.
- [12] T. Xu and I. Darwazeh, "A joint waveform and precoding design for non-orthogonal multicarrier signals," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, San Francisco, CA, USA, Mar. 2017, pp. 1–6.
- [13] Y. Hama and H. Ochiai, "On the achievable spectral efficiency of non-orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 71, no. 11, pp. 6246–6257, Nov. 2023.
- [14] S.-G. Choi, S.-H. Seo, K.-C. Tong, Y.-G. Jung, M.-H. Choi, and H.-K. Song, "Optimizing MIMO performance via the integration of FTN and RIS," in *Computer and Communication Engineering, CCCE 2025*, F. Neri, Z. Jiang, H. R. Karimi, and P. Mercorelli, Eds., Commun. Comput. Inf. Sci., vol. 2642. Cham, Switzerland: Springer, 2026, pp. 55–66, doi: 10.1007/978-3-032-06757-9_6.
- [15] C. Zhang, B. K. Ng, H. Xu, C.-T. Lam, and H. Yanikomeroglu, "RIS-empowered OTFS modulation with faster-than-Nyquist signaling in high-mobility wireless communications," arXiv:2512.20332, 2025, doi: 10.48550/arXiv.2512.20332.
- [16] Q. Wu and R. Zhang, "Beamforming optimization for wireless network aided by intelligent reflecting surface with discrete phase shifts," *IEEE Trans. Commun.*, vol. 68, no. 3, pp. 1838–1851, Mar. 2020.
- [17] J. An et al., "Stacked intelligent metasurface-aided MIMO transceiver design," *IEEE Wireless Commun.*, vol. 31, no. 4, pp. 123–131, Aug. 2024.
- [18] J. An et al., "Stacked intelligent metasurfaces for efficient holographic MIMO communications in 6G," *IEEE J. Sel. Areas Commun.*, vol. 41, no. 8, pp. 2380–2396, Aug. 2023.
- [19] J. An, M. Di Renzo, M. Debbah, H. V. Poor, and C. Yuen, "Stacked intelligent metasurfaces for multiuser downlink beamforming in the wave domain," *IEEE Trans. Wireless Commun.*, vol. 24, no. 7, pp. 5525–5538, Jul. 2025.
- [20] I. Choi and S. Kojima, "Stacked intelligent metasurfaces-based joint source-channel coding for image transmissions," *IEEE Open J. Commun. Soc.*, vol. 7, pp. 2825–2837, 2026.
- [21] K. Tamura, S. Kojima, K. Komatsu, J. Cha, and C.-J. Ahn, "Spectrally efficient frequency division multiplexing with QRM-MLD for Rician fading channel," in *Proc. 2023 International Conference on Advanced Technologies for Communications (ATC)*, Da Nang, Vietnam, 2023, pp. 128–133.
- [22] E. Björnson and L. Sanguinetti, "Rayleigh fading modeling and channel hardening for reconfigurable intelligent surfaces," *IEEE Wireless Commun. Lett.*, vol. 10, no. 4, pp. 830–834, Apr. 2021.
- [23] W. Ozan, R. Grammenos, and I. Darwazeh, "Zero padding or cyclic prefix: Evaluation for non-orthogonal signals," *IEEE Communications Letters*, vol. 24, no. 3, pp. 690–694, Mar. 2020.
- [24] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge University Press, 2004.
- [25] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, "Weighted sum-rate maximization for reconfigurable intelligent surface aided wireless networks," *IEEE Trans. Wireless Commun.*, vol. 19, no. 5, pp. 3064–3076, May 2020.