

Comparisons for Uplink PD-NOMA, GDMA and HDS-CDMA Systems

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Abstract—This paper presents a comparative study of uplink non-orthogonal multiple access (NOMA) systems, progressing from uncoded single-resource detection to uncoded and coded multi-resource reception. First, uncoded single-resource power-domain NOMA (PD-NOMA) with soft successive interference cancellation (SIC) is compared with gain division multiple access (GDMA) to clarify the impact of different user-separation principles. Second, PD-NOMA is extended to a two-resource / multi-resource vector observation model, where MMSE-SIC is evaluated against high density signature CDMA (HDS-CDMA) under uncoded transmission. The results show that HDS-CDMA achieves better uncoded BER performance, especially when users have similar large-scale fading coefficients. Finally, a coded multi-resource PD-NOMA receiver based on MMSE-SIC with iterative detection and decoding (MMSE-SIC-IDD) is developed by incorporating decoder-aided soft interference cancellation, variance-aware MMSE filtering, and outer detector-decoder feedback.

Index Terms—GDMA, HDS-CDMA, IDD, PD-NOMA, SIC, MMSE-SIC.

I. INTRODUCTION

In uplink NOMA, the receiver structure is often the key factor that determines both reliability and practicality. Different NOMA schemes rely on different user-separation principles. Gain division multiple access (GDMA) separates users by exploiting their complex channel gains and is typically associated with joint MAP or ML detection [1]. Power-domain NOMA (PD-NOMA) mainly relies on received-power disparity and usually adopts successive interference cancellation (SIC) [2], [3]. Code-domain schemes such as SCMA, low density signature CDMA (LDS-CDMA) and HDS-CDMA exploit sparse or structured signatures for user separation [4], [5]. Among these schemes, multi-resource PD-NOMA can be viewed as an intermediate structure: it preserves the successive-cancellation nature of PD-NOMA, while extending scalar single-resource detection to vector-domain processing across multiple shared resources.

Although SIC, MMSE-based receivers, and iterative detection and decoding (IDD) have been widely studied, it remains useful to provide a side-by-side comparison that answers three practical questions: (i) how single-resource PD-NOMA compares with GDMA under uncoded transmission, (ii) how a two-/multi-resource PD-NOMA receiver with MMSE-SIC performs relative to HDS-CDMA under different large-scale

fading conditions, and (iii) how channel coding and outer detector-decoder iterations change this comparison in a coded multi-resource receiver.

Contributions:

- We compare *uncoded single-resource* PD-NOMA (soft SIC) and GDMA to highlight their separation mechanisms and BER behavior.
- We evaluate *uncoded two-/multi-resource* PD-NOMA with MMSE-SIC and HDS-CDMA with ML detection, and show how the performance gap depends on equal and unequal large-scale fading conditions.
- We develop a *coded multi-resource* PD-NOMA receiver using MMSE-SIC-IDD, incorporating decoder-aided soft cancellation, variance-aware MMSE filtering, and outer detector-decoder feedback.

The remainder of this paper is organized as follows. Section II introduces the single-resource systems and the corresponding detectors. Section III presents the multi-resource PD-NOMA model and the MMSE-SIC detector. Section IV extends the receiver to coded MMSE-SIC-IDD. Finally, Section V concludes the paper.

II. SINGLE RESOURCE SYSTEMS

This section establishes the uncoded single-resource (scalar) uplink model and serves as the first stage of our comparison: PD-NOMA with soft SIC versus GDMA. This setting isolates the user-separation mechanism before multi-resource processing is introduced.

For an uplink communication system, U users transmit simultaneously over the same resource, where a resource can be a frequency band, a time slot, or a signature. The received signal at the base station is

$$y = \sum_{k=1}^U \sqrt{P_k \beta_k} h_k x_k + n, \quad (1)$$

where P_k is the transmit power, β_k is the large-scale fading coefficient, h_k is the small-scale Rayleigh flat-fading coefficient, and $n \sim \mathcal{CN}(0, N_0)$ is complex AWGN. Equivalently, the noise variance per dimension is $\sigma^2 = N_0/2$. In this work, for simplicity, $P_k = 1$ is assumed unless otherwise stated.

A. PD-NOMA with soft SIC detection

In PD-NOMA, users are mainly separated by their received-power difference. The instantaneous received-power metric of user k can be written as

$$\rho_k = P_k \beta_k |h_k|^2 \triangleq |g_k|^2, \quad g_k = \sqrt{P_k \beta_k} h_k. \quad (2)$$

Let the SIC detection order be $\pi = \{\pi(1), \pi(2), \dots, \pi(U)\}$, where $\pi(i)$ denotes the user index detected at the i^{th} SIC stage. For power-based ordering, the order is usually determined such that $\rho_{\pi(1)} \geq \rho_{\pi(2)} \geq \dots \geq \rho_{\pi(U)}$. The renumbering of user index is then processed by following the orders. For simplicity, we assume the indices of users in (1) have been renumbering.

At the first SIC stage, the LLR of x_1 , L_{x_1} , is obtained by regarding signals from other users as approximately Gaussian in (1) and the soft estimate is denoted as $\hat{s}_1$. At the $(i+1)^{\text{th}}$ SIC stage, the LLR of x_{i+1} , $L_{x_{i+1}}$, is obtained by a residual signal $y_{(i+1)}$ which is obtained by subtracting the contribution of the soft estimate $\hat{s}_i$ in the i^{th} stage and regarding signals from other users as Gaussian. Therefore, if $y_{(1)} = y$,

$$y_{(i+1)} = y_{(i)} - g_i \hat{s}_i, \quad (3)$$

For BPSK modulation, the log likelihood ratio (LLR) of user x_i can be simplified as

$$L_{x_i} = \log \frac{p(y_{(i)} | x_i = +1)}{p(y_{(i)} | x_i = -1)} = \frac{4\Re\{g_i^* y_{(i)}\}}{\sigma_{\text{eff},(i)}^2}. \quad (4)$$

The soft estimate of the detected symbol given $y_{(i)}$ is expressed [6] by

$$\hat{s}_i = \mathbb{E}[x_i | L_{x_i}] = \tanh\left(\frac{L_{x_i}}{2}\right). \quad (5)$$

and the conditional variance is obtained [6] as

$$\sigma_{\hat{s}_i}^2 = 1 - \tanh^2\left(\frac{L_{x_i}}{2}\right). \quad (6)$$

Therefore, the effective noise variance is

$$\sigma_{\text{eff},(i)}^2 = 2\sigma^2 + \sum_{j=i+1}^U |g_j|^2 + \sum_{j=1}^{i-1} |g_j|^2 \sigma_{\hat{s}_j}^2. \quad (7)$$

This procedure is repeated until all users are detected.

B. GDMA detection

In GDMA, users are distinguished by their distinct channel gains. At the receiver, the superimposed signal is first used to calculate *a posteriori* probabilities (APP) of all possible multiuser symbol combinations, and the LLR of each user is then obtained accordingly [1]. For BPSK modulation, $x_k \in \{+1, -1\}$. Therefore, there are 2^U possible superimposed signal levels, denoted by the set

$$\mathcal{S} = \{s_0, s_1, \dots, s_{2^U-1}\}, \quad (8)$$

where $s_l \in \mathcal{S}$ contributes at the signal part in (1).

If the signals in $\mathcal{S}$ are equally probable, the receiver computes the *a posteriori* probabilities (APP) of each possible superimposed signal $s_l \in \mathcal{S}$ by

$$P(s_l | y) \propto \exp\left(-\frac{|y - s_l|^2}{2\sigma^2}\right), \quad (9)$$

and then calculates the log-likelihood ratio (LLR) of user k by

$$L_k = \log \frac{\sum_{x_k=+1} \exp\left(-\frac{|y - s_l|^2}{2\sigma^2}\right)}{\sum_{x_k=-1} \exp\left(-\frac{|y - s_l|^2}{2\sigma^2}\right)}. \quad (10)$$

Unlike single-resource PD-NOMA with soft SIC detection, GDMA does joint detection. It exploits both the magnitude and phase of the users' channel gains to distinguish users in the superimposed signal. As a result, GDMA generally outperforms SIC-based detection as the number of users increases, particularly when the users can be effectively separated by their channel-gain differences, at the expense of higher detection complexity.

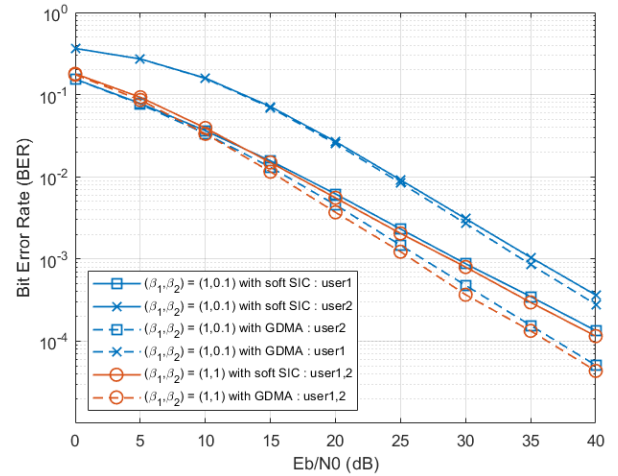


Fig. 1. Users' BERs for single resource systems. The cases of $(\beta_1, \beta_2) = (1, 1)$ and $(1, 0.1)$ are indicated by red lines and blue lines, respectively.

In this work, E_b denotes the average transmit energy per information bit per user, normalized by the number of shared resources. Perfect channel state information (CSI) is assumed at the receiver throughout this work.

Fig. 1 shows the BER comparison between soft SIC and GDMA for two-user transmission under $(\beta_1, \beta_2) = (1, 1)$ and $(1, 0.1)$. In this figure, the dashed curves indicate GDMA, while the solid curves indicate soft SIC. It is observed that GDMA consistently achieves lower BER than soft SIC in both large-scale fading cases. Furthermore, as the number of users increases, GDMA still maintains favorable performance [7], whereas the performance of soft SIC is expected to become more limited due to its successive cancellation structure and sensitivity to error propagation.

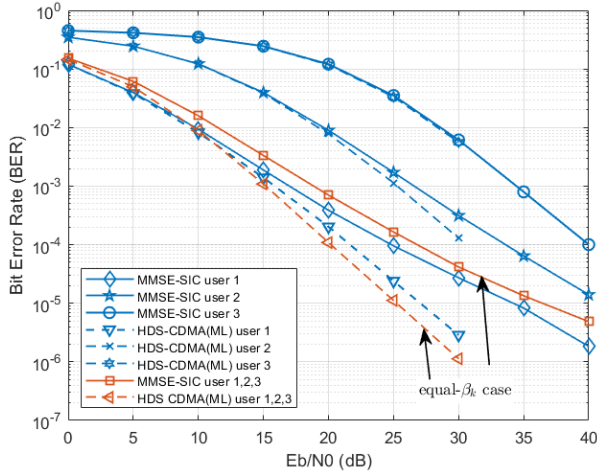


Fig. 2. Users' BERs for uncoded two-resource systems. The cases of $(\beta_1, \beta_2, \beta_3) = (1, 1, 1)$ and $(1, 0.1, 0.01)$ are indicated by red lines and blue lines, respectively.

III. MULTI-RESOURCE PD-NOMA

This section forms the second comparison stage by extending PD-NOMA to a vector observation model across multiple shared resources. While the formulation is general for R resources, our key uncoded comparisons emphasize the representative two-resource case.

In this paper, multi-resource PD-NOMA refers to an architecture in which each user can transmit over multiple shared resources. The multi-resource PD-NOMA forms a receiving model:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (11)$$

where $\mathbf{y} \in \mathbb{C}^{R \times 1}$ are the received signals over R resources, $\mathbf{x} \in \mathbb{C}^{U \times 1}$ are the transmitted signals of U users, $\mathbf{H} \in \mathbb{C}^{R \times U}$ represents the channel matrix in which the $(r, k)^{th}$ element is $\sqrt{P_k} \beta_k h_{r,k}$, $h_{r,k}$ denotes the Rayleigh flat-fading gain of user k on resource r , and $\mathbf{n}$ denotes complex AWGN.

We employ MMSE-SIC detection for (11). The resulting filter structure is similar to those widely used in multi-antenna systems [8], [9] and pattern division multiple access (PDMA) receivers [10].

A. MMSE-SIC Detection

For the multi-resource model of (11), consider user k be detected. The MMSE filter is designed by following [10, Eqs. (6), (7)] with the coefficient

$$\mathbf{w}_k^H = \mathbf{h}_k^H (\mathbf{H}\mathbf{H}^H + 2\sigma^2 \mathbf{I})^{-1}, \quad (12)$$

where $\mathbf{h}_k$ is the k^{th} column of $\mathbf{H}$. The filter output can be formulated as

$$z_k = \mathbf{w}_k^H \mathbf{y} = \mathbf{w}_k^H \mathbf{h}_k x_k + \sum_{j \neq k} \mathbf{w}_k^H \mathbf{h}_j x_j + \mathbf{w}_k^H \mathbf{n}. \quad (13)$$

In (13), the effective channel gain is $g_k = \mathbf{w}_k^H \mathbf{h}_k$. The effective variance introduced by the residual interferences and noise in (13) is obtained by

$$\sigma_{k,\text{eff}}^2 = \sum_{j \neq k} |\mathbf{w}_k^H \mathbf{h}_j|^2 + 2\sigma^2 \|\mathbf{w}_k^H\|^2. \quad (14)$$

To determine the successive detection orders in the SIC process, post-SINR ordering [9] is adopted to achieve better performance. For user k , the post-SINR is

$$\gamma_k^{\text{post}} = \frac{|\mathbf{w}_k^H \mathbf{h}_k|^2}{\sigma_{k,\text{eff}}^2}. \quad (15)$$

This ordering reflects the quality for each user after MMSE filtering, including the residual interference and noise enhancement in (14). The resulting post-SINR metric requires channel state information (CSI) including both their magnitudes and phases, which are generally preferred in reducing SIC error propagation. After determining the SIC orders and then re-numbering the user indices, we assume $\gamma_1^{\text{post}} \geq \dots \geq \gamma_U^{\text{post}}$ for simplicity. Similar to (3) in section II-A, at the first SIC stage, the LLR L_{x_1} is obtained by calculating (13) and then its soft estimate $\hat{s}_1$ is also obtained as the aforementioned. At the $(i+1)^{th}$ stage, the SIC detector performs the subtraction by setting

$$\mathbf{y}_{(i+1)} = \mathbf{y}_{(i)} - \mathbf{h}_i \hat{s}_i \quad (16)$$

with $\mathbf{y}_{(1)} = \mathbf{y}$, where $\mathbf{y}_{(i)}$ denotes the residual signal at stage i . The corresponding column of $\mathbf{H}$ in (11) is then removed by setting

$$\mathbf{H}_{(i+1)} = [\mathbf{h}_{i+1} \ \mathbf{h}_{i+2} \ \dots \ \mathbf{h}_U], \quad (17)$$

where $\mathbf{H}_{(1)} = \mathbf{H}$. The process then goes back to re-calculate the coefficient $\mathbf{w}_{i+1}^H$ by using (12), the corresponding filter output and the effective variance of the residual interferences and noise can be derived as

$$z_i = \mathbf{w}_i^H \mathbf{y}_{(i)} = \mathbf{w}_i^H \mathbf{h}_i x_i + \sum_{j>i} \mathbf{w}_i^H \mathbf{h}_j x_j + \mathbf{w}_i^H \mathbf{n}. \quad (18)$$

$$\sigma_{i,\text{eff}}^2 = \sum_{j>i} |\mathbf{w}_i^H \mathbf{h}_j|^2 + 2\sigma^2 \|\mathbf{w}_i^H\|^2. \quad (19)$$

Following similar procedures in section II-A, the LLR of x_{i+1} and its soft estimate $\hat{s}_{i+1}$ can be obtained. This procedure is repeated until all users are detected.

In addition to post-SINR ordering, received power-based ordering is a simpler alternative by sorting users only according to their channel magnitudes or received powers. Since this metric does not fully capture the filtering interferences, it is generally inferior to post-SINR ordering.

Fig. 2 illustrates the BER results of BPSK transmission for a three-user two-resource system over independent Rayleigh flat fading channels. The comparisons between MMSE-SIC and HDS-CDMA with ML detection [5] are conducted under $(\beta_1, \beta_2, \beta_3) = (1, 0.1, 0.01)$ and $(1, 1, 1)$. In this figure, the blue curves indicate the unequal- β_k case, while the red curves indicate the equal- β_k case. The solid curves denote MMSE-SIC detection and the dashed curves denote HDS-CDMA

At the SIC stage $i > 1$, the detection of user i is benefitted from the FEC outputs of users $1, 2, \dots, i-1$ and the outer information of users $i+1, \dots, U$. Because the residual vector for detecting user i is constructed by

$$\mathbf{y}_{(i)}^{(I_{out})} = \mathbf{y} - \sum_{j=1}^{i-1} \mathbf{h}_j \hat{\mathbf{s}}_j^d - \sum_{j=i+1}^U \mathbf{h}_j \hat{\mathbf{s}}_j^{(I_{out})}. \quad (25)$$

Similar to the procedures in (16), (17), (18) by using the downsized channel matrix and downsized variance matrix, the corresponding filter output can be obtained, and the variance of residual interference and noise is

$$\sigma_{i,\text{eff}}^2(I_{out}) = \sum_{j>i} \sigma_{\hat{\mathbf{s}}_j}^2(I_{out}) |\tilde{\mathbf{w}}_i^H \mathbf{h}_j|^2 + 2\sigma^2 \|\tilde{\mathbf{w}}_i^H\|^2. \quad (26)$$

Compared with uncoded MMSE-SIC, the coded receiver integrates decoder-aided soft cancellation, variance-aware MMSE filtering and detector-decoder outer iterations.

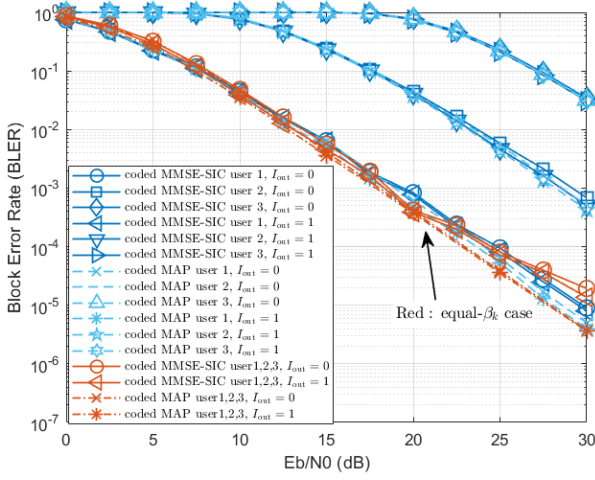


Fig. 4. Users' BLERs for coded two-resource systems. The cases of $(\beta_1, \beta_2, \beta_3) = (1, 1, 1)$ and $(1, 0.1, 0.01)$ are indicated by red lines and blue lines, respectively.

Fig. 4 shows the BLER performance of the coded two-resource system for three users using a BPSK modulated (1008, 504) LDPC code. The multiple access channel is assumed block faded. That is, $\mathbf{H}_1 = \dots = \mathbf{H}_L$. The solid curves denote coded MMSE-SIC-IDD, while the dashed curves denote coded HDS-CDMA with MAP-based detection. For the unequal- β_k case, coded MMSE-SIC-IDD performs close to coded MAP-based detection for all three users, indicating that decoder-aided soft cancellation effectively reduces residual interference. For the equal- β_k case, a similar tendency of BLER is observed, while coded HDS-CDMA is slightly better in the high-SNR region. Under the considered block-fading setting, only a limited additional gain is observed from increasing the number of outer iterations.

V. CONCLUSIONS

This paper presented uncoded and coded comparisons for uplink NOMA systems with single-resource or multi-resource

reception. For the single-resource case, GDMA with joint MAP-detection performs better than PD-NOMA with soft SIC because GDMA exploits the channel information but PD-NOMA encounters SIC error propagation. For the uncoded multi-resource case, MMSE-SIC improves the performance by introducing soft estimates in SIC, while HDS-CDMA with ML detection still achieves better BER performance due to its joint multiuser detection capability. For the coded multi-resource case, the MMSE-SIC-IDD receiver adopts the channel coding to decrease the error propagation in the SIC stage and integrates decoder-aided soft cancellation, variance-aware MMSE filtering and outer detector-decoder feedback, achieving BLER performance close to the coded HDS-CDMA with MAP-based detection. However, MMSE-SIC and MMSE-SIC-IDD offer lower-complexity alternatives for multi-resource NOMA systems. Future works will extend the comparison to the cases of more users sharing more resources and higher modulation orders. Fast fading cases are also required to be investigated.

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