

Overloaded MIMO Assisted Bi-Directional Communication Based on Physical Layer Network Coding With Resource Allocation

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Abstract—This paper proposes overloaded multiple-input-multiple-input (MIMO) assisted bi-directional communication with resource allocation in 2-hop relay networks that achieves higher throughput by utilizing more relays. Non-linear precoding for overloaded MIMO spatial multiplexing is applied in the networks, because the precoding can increase the number of the relays in spite of that of the terminal antennas. The proposed communication can increase the throughput as more relays are employed. Furthermore, this paper proposes resource allocation for the communication. The throughput of the proposed communication is evaluated by computer simulation. Even when only 2 antennas are placed on the terminals with the quaternary phase shift keying (QPSK), the proposed bi-directional communication achieves 14 bits/Hz at the E_b/N_0 of about 30 dB in the 2-hop relay network based on the physical layer network coding.

Index Terms—Network coding, Overloaded MIMO, Precoding, Resource allocation

I. INTRODUCTION

Higher frequency bands such as millimeter wave bands have been considered for cellular systems and IEEE wireless local area networks (WLANs). As is well known, radio signals are more attenuated in wireless channels as the carrier frequency is raised. To mitigate the signal attenuation, wireless multihop communication has been considered not only in higher frequency bands but also the microwave bands. However, wireless multihop communication reduces transmission efficiency, i.e., frequency utilization efficiency. When communication becomes bi-directional, the frequency utilization efficiency is further reduced in wireless relay networks. Physical layer network coding (PLNC) is known to improve the frequency utilization efficiency in bi-directional wireless relay networks [1]–[6]. On the other hand, since the frequency utilization efficiency is one of the most important issues, many other techniques have also been considered, such as multiple-input-multiple-input (MIMO) spatial multiplexing [7]–[9], resource allocation [10], [11], scheduling [12], [13], and so on. Overloaded MIMO spatial multiplexing is also recognized to increase the throughput [14]–[16]. MIMO spatial multiplexing has been investigated in bi-directional wireless relay network for raising the frequency

utilization efficiency [17]–[20]. Quadrature amplitude modulations (QAMs) have also been considered in multi-hop wireless relays with physical layer network coding for enhancing the throughput [21]. Scheduling has also been considered in the networks [22].

This paper proposes overloaded MIMO-assisted bi-directional communication with resource allocation in 2-hop relay networks that achieves higher throughput by utilizing more relays. Non-linear precoding for overloaded MIMO is introduced in the proposed communication as well as the physical layer network coding. The precoding enables the number of the relays in use to exceed that of the transmit antennas on the terminals. Because one signal stream is delivered to every relay in use, the throughput can be increased as more relays are utilized. For the network, we propose a new resource allocation technique that selects relays. The proposed resource allocation enables the proposed bi-directional communication to increase the throughput proportional to the signal to noise power ratio (SNR).

Throughout the paper, $\Re[c]$ and $\Im[c]$ represent a real part and an imaginary part of a complex number c , respectively. Superscript T and H indicate transpose and Hermitian transpose of a matrix or a vector, respectively. $\mathbf{A}^{-1}$, $\text{tr}[\mathbf{A}]$, and $(\mathbf{A})_{m,n}$ indicate inversion, a trace, and an (m,n) -entry of a matrix $\mathbf{A}$. $\text{diag}[\mathbf{V}]$ indicates a diagonal matrix with a vector $\mathbf{V}$ in the diagonal positions. $\mathbb{E}[c]$ and $\lfloor r \rfloor$ denote the ensemble average of c and the biggest integer less than r . Let $a \in \mathbb{R}$ and $b \in \mathbb{R}$ represent real numbers, $\text{mod}[a,b]$ indicates the modulo function defined as $\text{mod}[a,b] = a - \text{int}[\frac{a}{b}] \cdot b$ where $\text{int}[\bullet]$ indicates a function where an input is rounded down to the nearest integer of an input signal.

II. SYSTEM MODEL

We assume that two terminals with N_T antennas exchange their own information via relays with one antenna. The number of the relays is denoted by N_R . The two terminals are named as terminal A and terminal B, respectively. Information bit stream is fed to an encoder and the encoder output bit stream is provided to a quaternary phase shift keying (QPSK) modulator via an interleaver at the two terminals. The modulator outputs a modulation signal $d_A(k) \in \mathbb{C}$ where $k \in \mathbb{Z}$ denotes a stream

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index. Let $\mathbf{D}_A \in \mathbb{C}^{N_S}$ represent a modulation signal vector defined as $\mathbf{D}_A = (d_A(1) \cdots d_A(N_S))^T$ where $N_S \in \mathbb{Z}$ represents the number of the spatially multiplexed signal streams, the modulation signal vector $\mathbf{D}_A$ is precoded in a manner described in the following section, and sent for the relays. The modulation signal vector $\mathbf{D}_B \in \mathbb{C}^{N_S}$ sent from the terminal B can be defined in the same manner. When a precoded signal vector is denoted as $\mathbf{X}_A \in \mathbb{C}^{N_T}$, the vector is defined as $\mathbf{X}_A = [x_A(1) \cdots x_A(N_T)]^T$ where $x_A(k) \in \mathbb{C}$ represents a transmission signal sent from the k th antenna on the terminal A. The transmission signal vector $\mathbf{X}_B \in \mathbb{C}^{N_T}$ from the terminal B can be also defined in the same manner. When the terminal A and the terminal B simultaneously send their packets containing the precoded signals, those packets are received at all the relays in the first slot. Let $y_R(k) \in \mathbb{C}$ denote a received signal at the k th relay, the received signal is written as,

$$y_R(k) = \mathbf{h}_{A,k} \mathbf{X}_A + \mathbf{h}_{B,k} \mathbf{X}_B + \xi(k). \quad (1)$$

In (1), $\mathbf{h}_{\Omega,k} \in \mathbb{C}^{1 \times N_T}$ $\Omega = A$ or B and $\xi(k) \in \mathbb{C}$ represent a channel vector between the k th relay and the terminal Ω and an additive white Gaussian noise (AWGN) at the k th relay.

The physical layer network coding applies rate-one coding to the received signal $y_R(k)$. Let $\oplus$ indicate rate-one encoding, the rate-one encoded signal $d_R(k) \in \mathbb{C}$ is defined as $d_R(k) = d_A(k) \oplus d_B(k)$. So called Exclusive OR is used for rate-one encoding in this paper. The encoded signal can be estimated from the received signal [19], [20].

$$\begin{aligned} \bar{d}_R(k) &= \arg \max_{d_R(k)} p(d_R(k) | y_R(k)) \\ &= \arg \max_{d_R(k)=d_A(k) \oplus d_B(k)} p(d_A(k) \oplus d_B(k) | y_R(k)) \end{aligned} \quad (2)$$

In (2), $\bar{d}_R(k)$ and $p(a|b)$ denote an estimate of the rate-one encoded signal and a conditional probability that an event a happens when an event b occurred. The encoded signals are fed to the channel decoder at every relay and the decoded signals are again encoded. The encoded signals are fed to the QPSK modulator via the interleaver, and the modulated signals are broadcast for the terminals in the second slot. Let $x_R(k) \in \mathbb{C}$ represent a QPSK modulated signal fed by the QPSK modulator at the k th relay, when some relays send their own encoded signals simultaneously, the received signal vector $\mathbf{Y}_\Omega \in \mathbb{C}^{N_T}$ at terminal Ω is written as,

$$\begin{aligned} \mathbf{Y}_\Omega &= \sum_{k \in \mathcal{B}_m^{(N_S)}} \mathbf{h}_{\Omega,k}^H x_R(k) + \xi_\Omega \\ &= \mathbf{H}_{\Omega,m}^H \mathbf{X}_R + \xi_\Omega. \end{aligned} \quad (3)$$

In the above equation, $\xi_\Omega \in \mathbb{C}^{N_T}$ and $\mathcal{B}_m^{(N_S)}$ denote an AWGN vector at terminal Ω and a set that contains N_S indexes of the relays transmitting the signals in the second slot, where m represents a set index. In addition, $\mathbf{H}_{\Omega,m} \in \mathbb{C}^{N_S \times N_T}$ and $\mathbf{X}_R \in \mathbb{C}^{N_S}$ represent a composite channel matrix and a composite transmission signal vector, which are defined as $\mathbf{H}_{\Omega,m}^H = [\mathbf{h}_{\Omega,m_1}^H \cdots \mathbf{h}_{\Omega,m_{N_S}}^H]$ and $\mathbf{X}_R = [x_R(m_1) \cdots x_R(m_{N_S})]^T$,

where $m_i \in \mathbb{Z}$ indicates a relay index in the set $\mathcal{B}_m^{(N_S)}$. Since N_R relays are located around the two terminals, the number of the sets $\mathcal{B}_m^{(N_S)}$ is $N_R C_{N_S}$, i.e., $m = 1, \dots, N_R C_{N_S}$. Because the system defined in (3) is regarded as one of overloaded MIMO channels, non-linear detectors have to be applied, such as the maximum likelihood detector (MLD). When the decoder outputs an estimate of the rate-one encoded signal $\bar{d}_R(k)$ at the terminal Ω , the transmission signal sent from the other terminals can be estimated with the signal $d_\Omega(k)$ sent from the own terminal Ω , i.e., $\bar{d}_{\bar{\Omega}}(k) = \bar{d}_R(k) \oplus d_\Omega(k)$, where $\bar{d}_{\bar{\Omega}}(k)$ represents an estimate of the signal sent from the other terminal $\bar{\Omega}$. As is described above, the time division duplex (TDD) is used in the 2-hop network to exchange the information between the terminals.

III. OVERLOADED MIMO ASSISTED BI-DIRECTIONAL COMMUNICATION

We assume that the number of the relays N_R is more than that of the antennas on the terminals N_T , i.e., $N_R > N_T$. Because the terminals can't always deliver the signals to all the relays with high quality, the relays to forward the signals should be appropriately selected.

A. Precoding For Throughput Enhancement

For the performance enhancement, the terminals direct the beams to the relays. In order to be able to employ any number of the relays despite of the number of the transmit antennas, we use overloaded MIMO precoding. For achieving superior transmission performance in the first slot, the lattice reduction is introduced to the overloaded MIMO precoding as follows [23].

$$\left(\frac{\mathbf{H}_{\Omega,\hat{m}}^H}{\sqrt{\gamma} \mathbf{I}_{N_S}} \right) \mathbf{T}_{\Omega,\hat{m}} = \mathbf{Q}_{\Omega,\hat{m}} \mathbf{\Gamma}_{\Omega,\hat{m}} \mathbf{R}_{\Omega,\hat{m}} \quad (4)$$

In (4), $\mathbf{Q}_{\Omega,\hat{m}} \in \mathbb{C}^{(N_T+N_S) \times N_S}$, $\mathbf{\Gamma}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S \times N_S}$, $\mathbf{R}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S \times N_S}$, and $\mathbf{T}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S \times N_S}$ denote a semi-orthogonal matrix, a diagonal matrix, an upper unitriangular matrix, a unimodular matrix, respectively. When the lattice reduction is applied to the precoder, the minimum mean square error (MMSE) precoded transmission signal vector $\mathbf{X}_\Omega$ can be derived,

$$\begin{aligned} \mathbf{X}_\Omega &= g_{\Omega,\hat{m}} \mathbf{H}_{\Omega,\hat{m}}^H (\mathbf{H}_{\Omega,\hat{m}} \mathbf{H}_{\Omega,\hat{m}}^H + \gamma \mathbf{I}_{N_S})^{-1} \dot{\mathbf{D}}_\Omega \\ &= g_{\Omega,\hat{m}} \mathbf{F}_{\Omega,\hat{m}}^H \mathbf{v}_{\Omega,\hat{m}} \quad \Omega = A \text{ or } B. \end{aligned} \quad (5)$$

In (5), $\dot{\mathbf{D}}_\Omega \in \mathbb{C}^{N_S}$, $\mathbf{F}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S \times N_T}$, and $\mathbf{v}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S}$ represent a modulation signal vector with a Gaussian integer multiples vector, a feedforward filter, and a feedback filter output vector, respectively. The vector $\dot{\mathbf{D}}_\Omega$ is defined as $\dot{\mathbf{D}}_\Omega = \mathbf{D}_\Omega + \mathbf{T}_{\Omega,\hat{m}}^{-H} \mathbf{K}_{\Omega,\hat{m}} M_{\Omega,\hat{m}}$ where $\mathbf{D}_\Omega \in \mathbb{C}^{N_S}$, $\mathbf{K}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S}$, and $M_{\Omega,\hat{m}} \in \mathbb{R}$ denote a modulation signal vector, a Gaussian integer vector calculated through a modulo operation in (6), and a modulus defined as $M_{\Omega,\hat{m}} = 2 \left(1 + \frac{g_{\Omega,\hat{m}}}{g_{\bar{\Omega},\hat{m}}} \right)$ [20]. $g_{\Omega,\hat{m}} \in \mathbb{R}$ and $g_{\bar{\Omega},\hat{m}} \in \mathbb{R}$ denote normalization factors for the terminal Ω and the other terminal, respectively, which are defined below.

In addition, the feedforward filter $\mathbf{F}_{\Omega,\hat{m}}$ and the feedback filter output vector $\mathbf{v}_{\Omega,\hat{m}}$ are defined as follows.

$$\begin{aligned}\mathbf{F}_{\Omega,\hat{m}}^H &= \mathbf{H}_{\Omega,\hat{m}}^H \mathbf{T}_{\Omega,\hat{m}} \mathbf{R}_{\Omega,\hat{m}}^{-1} \mathbf{\Gamma}_{\Omega,\hat{m}}^{-2} \\ \mathbf{v}_{\Omega,\hat{m}} &= \text{mod} [\mathbf{T}_{\Omega,\hat{m}}^H \mathbf{D}_{\Omega} - \mathbf{B}_{\Omega,\hat{m}} \mathbf{v}_{\Omega,\hat{m}}, M_{\Omega,\hat{m}}]\end{aligned}\quad (6)$$

In (6), $\mathbf{B}_{\Omega,\hat{m}} \in \mathbb{C}^{N_S \times N_S}$ represents a feedback filter defined as $\mathbf{B}_{\Omega,\hat{m}} = \mathbf{I}_{N_S} - \mathbf{R}_{\Omega,\hat{m}}^H$, where $\mathbf{B}_{\Omega,\hat{m}}$ is a strictly upper triangular matrix. On the other hand, the normalization factor is defined in the following.

$$g_{\Omega,\hat{m}} = \sqrt{\frac{\sigma_d^2 N_S}{\text{tr} [\mathbf{F}_{\Omega,\hat{m}} \mathbf{F}_{\Omega,\hat{m}}^H \mathbf{\Phi}]}} \quad (7)$$

$\mathbf{\Phi} \in \mathbb{C}^{N_S \times N_S}$ in (7) represents a correlation matrix of the feedback filter output vector $\mathbf{v}_{\Omega,\hat{m}}$ defined as,

$$\begin{aligned}\mathbf{\Phi} &= \mathbb{E} [\mathbf{v}_{\Omega,\hat{m}} \mathbf{v}_{\Omega,\hat{m}}^H] \\ &\approx \text{diag} (\sigma_d^2 \frac{1}{6} M^2 \quad \dots \quad \frac{1}{6} M^2) \quad \Omega = \text{A or B.}\end{aligned}\quad (8)$$

In (8), $M \in \mathbb{R}$ is a constant. As is shown in (7), the transmission power is proportional to the number of the selected relays N_S .

B. Decoding and Forwarding at Relays

Because the transmission signals are precoded, the signals received at the relay are co-phased, which simplifies the signal detection as follows.

$$\begin{aligned}s_R(k) &= \text{mod} [y_R(k), M_{R,\hat{m}}] \\ &\approx g_{A,\hat{m}} d_A(k) + g_{B,\hat{m}} d_B(k) + \dot{\xi}(k)\end{aligned}\quad (9)$$

$s_R(k) \in \mathbb{R}$ and $M_{R,\hat{m}} \in \mathbb{R}$ represent a modulo function output signal and a modulus for the relay, the latter of which is defined as $M_{R,\hat{m}} = 2(g_{A,\hat{m}} + g_{B,\hat{m}})$ [20]. In addition, $\dot{\xi}(k) \in \mathbb{C}$ represents a noise component in the signal received at the k th relay, which is explicitly defined in the following section. As is described above, the signal $s_R(k)$ is provided to the rate-one encoder, and the encoder output signal is fed to the soft input channel decoder via the de-interleaver. The decoder output signals are re-encoded and the encoded signals are fed to the QPSK modulator via the interleaver. The QPSK modulator output signal $x_R(k)$ is broadcast for the terminals in the way described in (3).

C. SNR Estimation

To evaluate the SNR performance of the signal received at the relays, the noise component vector $\dot{\Xi}_m \in \mathbb{C}^{N_S}$ is defined as $\dot{\Xi}_m = [\dot{\xi}(m_1) \dots \dot{\xi}(m_{N_S})]^T$. The noise component vector $\dot{\Xi}_m$ can be approximately expressed in the following equation.

$$\begin{aligned}\dot{\Xi}_m &\approx -\gamma \left(g_{A,\hat{m}} \mathbf{T}_{A,\hat{m}} \mathbf{R}_{A,\hat{m}}^{-1} \mathbf{\Gamma}_{A,\hat{m}}^{-2} \mathbf{v}_{A,\hat{m}} \right. \\ &\quad \left. + g_{B,\hat{m}} \mathbf{T}_{B,\hat{m}} \mathbf{R}_{B,\hat{m}}^{-1} \mathbf{\Gamma}_{B,\hat{m}}^{-2} \mathbf{v}_{B,\hat{m}} \right) + \Xi\end{aligned}\quad (10)$$

$\Xi \in \mathbb{C}^{N_S}$ indicates an AWGN vector defined as $\Xi = [\xi(m_1) \dots \xi(m_{N_S})]^T$, and the correlation of the AWGN vector

can be written as $\mathbb{E} [\Xi \Xi^H] = 2\sigma^2 \mathbf{I}_{N_S}$. The correlation of the noise component vector can be derived as follows.

$$\begin{aligned}\mathbb{E} [\dot{\Xi}_m \dot{\Xi}_m^H] &= \gamma^2 g_{A,\hat{m}}^2 \mathbb{E} \left[\left| \mathbf{T}_{A,\hat{m}} \mathbf{R}_{A,\hat{m}}^{-1} \mathbf{\Gamma}_{A,\hat{m}}^{-2} \mathbf{v}_{A,\hat{m}} \right|^2 \right] \\ &\quad + \gamma^2 g_{B,\hat{m}}^2 \mathbb{E} \left[\left| \mathbf{T}_{B,\hat{m}} \mathbf{R}_{B,\hat{m}}^{-1} \mathbf{\Gamma}_{B,\hat{m}}^{-2} \mathbf{v}_{B,\hat{m}} \right|^2 \right] + 2\sigma^2 \mathbf{I}_{N_S} \\ &= \gamma^2 g_{A,\hat{m}}^2 \mathbf{T}_{A,\hat{m}} \mathbf{R}_{A,\hat{m}}^{-1} \mathbf{\Gamma}_{A,\hat{m}}^{-2} \mathbf{\Phi}_{A,\hat{m}} \mathbf{\Gamma}_{A,\hat{m}}^{-2} \mathbf{R}_{A,\hat{m}}^{-H} \mathbf{T}_{A,\hat{m}}^H \\ &\quad + \gamma^2 g_{B,\hat{m}}^2 \mathbf{T}_{B,\hat{m}} \mathbf{R}_{B,\hat{m}}^{-1} \mathbf{\Gamma}_{B,\hat{m}}^{-2} \mathbf{\Phi}_{B,\hat{m}} \mathbf{\Gamma}_{B,\hat{m}}^{-2} \mathbf{R}_{B,\hat{m}}^{-H} \mathbf{T}_{B,\hat{m}}^H \\ &\quad + 2\sigma^2 \mathbf{I}_{N_S}\end{aligned}\quad (11)$$

$\mathbf{\Phi}_{\Omega,\hat{m}}$ $\Omega = \text{A or B}$ in (11) indicates a correlation matrix of the feedback filter output vector $\mathbf{v}_{\Omega,\hat{m}}$ defined as follows.

$$\mathbf{\Phi}_{\Omega,\hat{m}} = \mathbb{E} [\mathbf{v}_{\Omega,\hat{m}} \mathbf{v}_{\Omega,\hat{m}}^H] \approx \text{diag} \left[\sigma_d^2 \frac{1}{6} M_{\Omega,\hat{m}}^2 \dots \frac{1}{6} M_{\Omega,\hat{m}}^2 \right] \quad (12)$$

The k th diagonal element of the correlation matrix $\mathbb{E} [\dot{\Xi}_m \dot{\Xi}_m^H]$ expresses the power of the noise $\dot{\xi}(k)$ written in (9), i.e., the noise component at the k th relay. Let $\eta(k)$ denote the SNR of the rate-one encoded signal fed to the channel decoder at the k th relay, the SNR can be derived in (13).

D. Throughput Maximization

As is written in (10), the noise component $\dot{\xi}(k)$ is expressed as a linear weight sum of the feedback filter output vectors $\mathbf{v}_{A,\hat{m}}$, $\mathbf{v}_{B,\hat{m}}$, and the AWGN $\xi(k)$. Based on the central limit theorem, the distribution of the noise component $\dot{\xi}(k)$ can be approximated with Gaussian distribution. Let $P_b(\eta(k)) \in \mathbb{R}$ indicate the bit error rate, the packet error rate $P_e(N_S) \in \mathbb{R}$ can be approximated as,

$$P_e(N_S) \approx 1 - \prod_{k=1}^{N_S} \left(1 - P_b(\eta(k)) \right)^{N_P}. \quad (15)$$

$N_P \in \mathbb{Z}$ represents the number of the symbols in a signal stream. With the packet error rate $P_e(N_S)$, the throughput can be approximated as follows.

$$S(N_S) \approx R_b N_b N_S (1 - P_e(N_S)) \quad (16)$$

$N_b \in \mathbb{Z}$, $R_b \in \mathbb{R}$, and $S(N_S) \in \mathbb{R}$ in (16) denote bits per symbol of a modulation scheme, a coding rate, and throughput of the 2-hop relaying when N_S relays are utilized for the communication. Because our objective is to maximize the throughput, we select the number of the relays that maximizes the throughput as,

$$\bar{N}_S = \arg \max_{N_S} [S(N_S)]. \quad (17)$$

$\bar{N}_S \in \mathbb{Z}$ represents the best number of the relays that maximizes the throughput. In the proposed communication, the best number of the relays is determined packet by packet as well as the best set.

$$\eta(k) = \frac{\sigma_d^2 g_{\Omega, \hat{m}}^2}{2\sigma^2 + \gamma^2 g_{A, \hat{m}}^2 \left(\mathbf{T}_{A, \hat{m}} \mathbf{R}_{A, \hat{m}}^{-1} \mathbf{\Gamma}_{A, \hat{m}}^{-2} \mathbf{\Phi}_{A, \hat{m}} \mathbf{\Gamma}_{A, \hat{m}}^{-2} \mathbf{R}_{A, \hat{m}}^{-H} \mathbf{T}_{A, \hat{m}}^H \right)_{k,k} + \gamma^2 g_{B, \hat{m}}^2 \left(\mathbf{T}_{B, \hat{m}} \mathbf{R}_{B, \hat{m}}^{-1} \mathbf{\Gamma}_{B, \hat{m}}^{-2} \mathbf{\Phi}_{B, \hat{m}} \mathbf{\Gamma}_{B, \hat{m}}^{-2} \mathbf{R}_{B, \hat{m}}^{-H} \mathbf{T}_{B, \hat{m}}^H \right)_{k,k}} \quad (13)$$

$$\Omega = \begin{cases} A & g_{A, \hat{m}} \leq g_{B, \hat{m}} \\ B & g_{A, \hat{m}} > g_{B, \hat{m}} \end{cases} \quad (14)$$

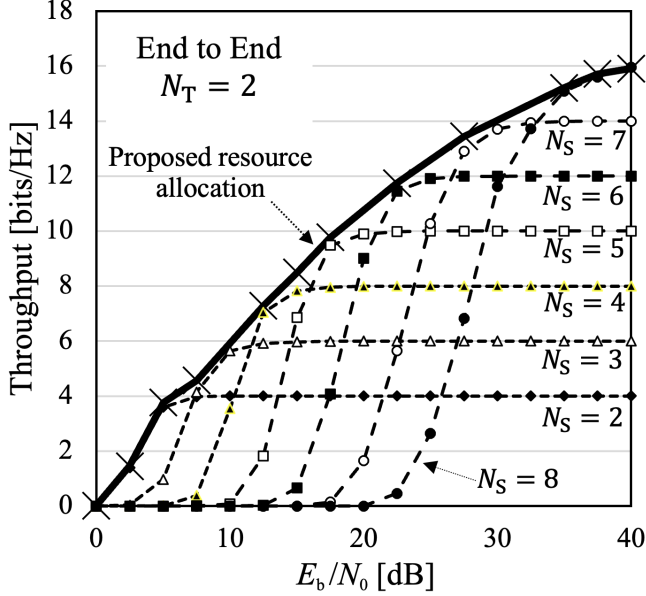


Fig. 1. End-to-End Throughput

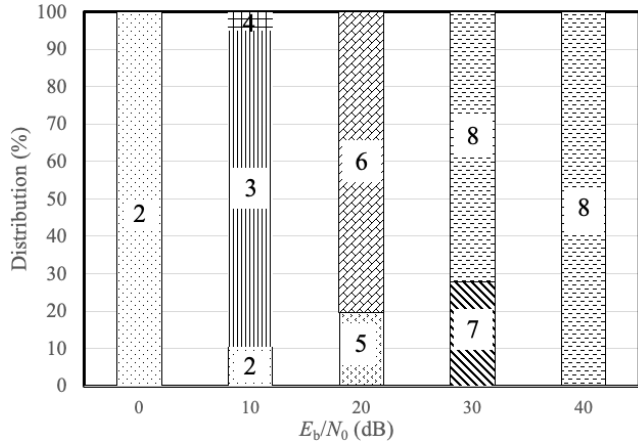


Fig. 2. Distribution of the number of relays

IV. SIMULATION

The performance of the proposed overloaded MIMO assisted bi-directional communication with resource allocation in a 2-hop relay network is evaluated by computer simulation. The physical layer network coding is employed in the communication. As is described, the QPSK is applied. The half rate convolutional code with constraint length of 3 is used, and the soft input Viterbi decoder is employed at the relays. The Lenstra–Lenstra–Lovász (LLL) algorithm is applied to the non-

TABLE I
SIMULATION PARAMETERS

Modulation	QPSK / Single Carrier
Precoding	Lattice reduction aided non-linear
Lattice reduction	LLL algorithm ($\delta = 0.75$) [28]
Packet length, N_P	1200 symbols
Channel model	i.i.d. Rayleigh fading
No. of terminal antennas, N_T	2
No. of relays, N_R	15
Relay selection range, N_S	2 ~ 8
Channel estimation	Perfect
Error correction code	Convolutional code ($R = \frac{1}{2}$, $K = 3$)
Decoder at relays	Soft input Viterbi algorithm
Decoder at terminals	Hard input Viterbi algorithm

linear precoder for the lattice reduction [25], [26]. On the other hand, the MLD is applied for the signal detection, and its output signals are fed to the hard input Viterbi decoder at the terminals in the second slot. The independent and identically distributed (i.i.d.) Rayleigh fading based on Jakes' model [27] is applied to all the channels between the antenna on the terminal and that on the relay. The number of the antennas on the terminal is fixed to 2, i.e., $N_T = 2$. The maximum number of the relays N_R is 15. Simulation parameters are listed in Table I.

A. Performance of Proposed Resource Allocation

Fig. 1 shows the end-to-end throughput in the 2-hop bi-directional wireless network, i.e., the sum rate of the bi-directional communications between the two terminals. In the performance evaluation, the frequency utilization reduction due to the TDD is taken into account. The horizontal axis and the vertical axis are the E_b/N_0 and the probability. The dotted lines show the throughput in the 2-hop network where only the proposed relay selection is applied; the number of the relays is fixed to N_S .

The solid line in Fig. 1 shows the end-to-end throughput in the proposed communication with the proposed resource allocation. The end-to-end throughput is just like an envelope of the dotted lines, which means that the proposed resource allocation makes the bi-directional communication achieve superior throughput performance despite of the E_b/N_0 . As the E_b/N_0 increases to about 40 dB, actually, the proposed resource allocation makes the throughput asymptotically reach to that with $N_S = 8$, because the proposed resource allocation deploys 8 relays at most. On the other hand, when the E_b/N_0 is reduced to less than 5 dB, the proposed resource allocation achieves the same throughput to the resource allocation with N_S of 2, respectively, because the proposed resource allocation has to employ at least 2 relays. As is shown in the figure,

the proposed communication achieves 14 bits/Hz at the E_b/N_0 of about 30 dB, even though only 2 antennas are installed on the terminals in the communication even if the QPSK is only applied.

B. Distribution of the number of the relays

The performance of the proposed resource allocation is analyzed with respect to the number of the relays used in the communication. Fig. 2 shows the distribution with respect to the number of the selected relays when the proposed resource allocation is applied. The horizontal axis is the E_b/N_0 and the vertical axis is the probability. In the figure, the numbers written in the shaded boxes represent the number of the relays user for the communication. When the E_b/N_0 is 0 dB, for instance, 2 relays are almost selected. When the E_b/N_0 increases, the probability that more relays are used in the network grows high. When the instantaneous E_b/N_0 surpasses the average E_b/N_0 , the number of the relays gets higher than the averaged number of the relays. This is the reason why the proposed resource allocation makes the proposed communication outperform that with the fixed number of the relays.

V. CONCLUSION

This paper has proposed overloaded MIMO assisted bi-directional communication with resource allocation in the 2-hop wireless network based on physical layer network coding. Overloaded MIMO non-linear precoding is applied to the terminals in order to be able to employ any number of the relays in spite of the terminal antennas. The physical layer network coding allows the terminals to exchange their information via the relays in two time slots for improving the throughput in the bi-directional communication. We propose new resource allocation for increasing the throughput.

The throughput performance is evaluated in a 2-hop wireless relay network where two terminals with 2 antennas exchange their signals via relays with 1 antenna. Even though the QPSK and the half rate convolutional code are used, the proposed communication can increase the throughput in proportion to the E_b/N_0 in virtue of the proposed resource allocation. Simulation shows that the proposed bi-directional communication achieves 14 bits/Hz at the E_b/N_0 of about 30 dB, even though the modulation scheme is fixed to the QPSK and only 2 antennas are installed on the terminals in the relay network where 15 relays are located around the two terminals.

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