

Decision-Feedback Channel Estimation for RIS-Assisted Communications

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Abstract—This paper presents a new family of phase-shift configurations based on Golay unitary matrices for reconfigurable intelligent surface (RIS)-assisted communications. Compared with conventional configurations derived from discrete Fourier transform or Hadamard matrices, the proposed configurations flexibly accommodate different numbers of reflecting elements and phase-shift constraints while largely preserving the orthogonality required for optimal least-squares channel estimation. When combined with decision-feedback channel estimation, the proposed configurations can further improve estimation accuracy and passive beamforming performance. These findings highlight the potential of the proposed configurations for practical deployment under hardware constraints.

I. INTRODUCTION

A passive reconfigurable intelligent surface (RIS) comprises an array of passive reflecting elements that can impose phase shifts on incident signals to redirect radio waves [1], [2]. Often, the RIS is deployed as a beamformer to maximize the signal-to-noise ratio (SNR) of the transmission from a base station (BS) to user equipments (UEs) [3], [4], which requires accurate channel state information (CSI) at the BS. However, as the RIS generally lacks signal processing capability, the cascaded channels between the BS and the UEs through each reflecting element must be estimated indirectly. The estimation of a large number of cascaded channels easily incurs substantial pilot-symbol, CSI-feedback, and computational overhead, making CSI acquisition in RIS-assisted communication systems challenging.

A common practice assumes channel reciprocity and leverages uplink (UL) cascaded channel estimation to support downlink passive beamforming [4], [5]. In this approach, the BS configures the

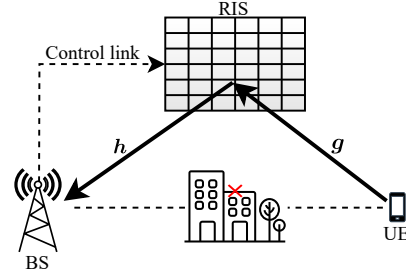


Fig. 1: Illustration of an RIS-assisted uplink communication system, where the direct link between the BS and the UE is blocked [8].

reflecting elements with different phase shifts during pilot transmission and estimates the cascaded UE-RIS-BS channel from the received pilot signals. To achieve optimal least-squares (LS) estimation, orthogonal RIS configurations based on discrete Fourier transform (DFT) or Hadamard matrices are widely considered [6], [7]. Practical variants, such as quantized DFT (QDFT) matrices and truncated Hadamard matrices (THMs), have also been proposed to accommodate restrictions on the allowable phase shifts and arbitrary numbers of passive reflecting elements at the RIS [4], [5].

Beyond pilot-only estimation, the detected UL data symbols can be exploited to further refine the cascaded channel estimate. Following the decision-feedback (DF) principle [9], the detected data symbols are treated as additional pilot symbols, thereby increasing the number of available observations for channel estimation. Its effectiveness, however, depends on both the detection reliability and the RIS configurations. Motivated by these considerations, we first introduce a new family of RIS configurations derived from Golay unitary matrices (GUMs) [10], which offers greater flexibility in satisfying the phase-shift constraints and the orthogonality requirement in optimal LS estimation. We then investigate DF-based channel estimation under different RIS configurations in terms of normalized mean-square error (NMSE) and detection error rate. In

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contrast to the hybrid RIS setting in [11], the purely passive RIS architecture studied in this work cannot exploit active sensing at the RIS, thereby imposing more stringent channel estimation requirements.

The remainder of this paper is organized as follows. Section II reviews the transmission model and practical RIS configurations. Section III presents the DF-based LS channel estimation schemes using GUM matrices. Numerical results are provided in Section IV, and Section V concludes the paper.

II. PRELIMINARIES

In this paper, we define $[K] \triangleq \{1, 2, \dots, K\}$ for any positive integer K . The notations $(\cdot)^\top$, $(\cdot)^\dagger$, and $(\cdot)^{-1}$ stand for transpose, conjugate transpose, and inverse, respectively. For a vector $\mathbf{a}$, $\text{diag}(\mathbf{a})$ denotes a diagonal matrix with $\mathbf{a}$ being its diagonal entries. The trace of a square matrix $\mathbf{A}$ is $\text{tr}(\mathbf{A})$. The expectation operator is denoted by $\mathbb{E}[\cdot]$, and $\mathcal{CN}(\mu, \sigma^2)$ denotes a circularly symmetric complex Gaussian distribution with mean μ and variance σ^2 .

A. System Model

Consider the RIS-assisted communication system in Fig. 1, where a single-antenna BS serves a single-antenna UE via an RIS with N passive elements due to blockage of the line-of-sight link. Let $x \in \mathbb{C}$ denote the modulated symbol of unit energy, and let E_s denote the average transmission energy. Define $\boldsymbol{\phi} \triangleq [e^{j\theta_1} \ e^{j\theta_2} \ \dots \ e^{j\theta_N}]^\top$ as the RIS phase-shift configuration with $\theta_i \in [0, 2\pi)$ for all $i \in [N]$. The received UL signal at the BS is given by

$$y = \sqrt{E_s} \mathbf{h}^\top \text{diag}(\boldsymbol{\phi}) \mathbf{g} x + n \quad (1a)$$

$$= \sqrt{E_s} \boldsymbol{\phi}^\top \text{diag}(\mathbf{h}) \mathbf{g} x + n \quad (1b)$$

$$= \sqrt{E_s} \boldsymbol{\phi}^\top \mathbf{c} x + n, \quad (1c)$$

where $\mathbf{g} \in \mathbb{C}^{N \times 1}$ and $\mathbf{h} \in \mathbb{C}^{N \times 1}$ are the UE-to-RIS and RIS-to-BS channel vectors, respectively, whose entries are independent and identically distributed (i.i.d.) according to $\mathcal{CN}(0, 1)$. Moreover, $n \in \mathbb{C}$ denotes a $\mathcal{CN}(0, \sigma^2)$ -distributed additive noise, and $\mathbf{c} \triangleq \text{diag}(\mathbf{h}) \mathbf{g} \in \mathbb{C}^{N \times 1}$ represents the so-called cascaded channel. Note that due to hardware constraints of RIS, each phase shift θ_i is often confined to a finite set $\mathcal{F} = \{0, \Delta\theta, \dots, (2^b - 1)\Delta\theta\}$ [4], where $\Delta\theta = \frac{2\pi}{2^b}$ for some positive integer b .

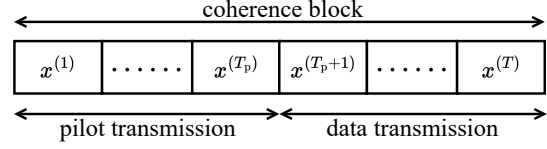


Fig. 2: Illustration of the structure of a coherence block.

B. Transmission Scheme

Suppose that the cascaded channel $\mathbf{c}$ has a coherence time of T symbols. Similar to conventional transmission schemes, each coherence block begins with a pilot phase, during which the UE transmits $T_p \leq T$ pilot symbols to enable the BS to estimate $\mathbf{c}$. The resulting channel estimate is then used for coherent detection of the remaining $T_d = T - T_p$ data symbols and, when needed, for downlink passive beamforming. An illustration is given in Fig. 2.

Letting the received signal at time t be

$$y^{(t)} = \sqrt{E_s} \boldsymbol{\phi}_t^\top \mathbf{c} x^{(t)} + n^{(t)}, \quad (2)$$

the first $\tau \leq T$ consecutive received signals can be expressed in matrix form:

$$\mathbf{y}_\tau = \sqrt{E_s} \text{diag}(\mathbf{x}_\tau) \boldsymbol{\Phi}_\tau \mathbf{c} + \mathbf{n}_\tau, \quad (3)$$

where $\mathbf{y}_\tau \triangleq [y^{(1)}, \dots, y^{(\tau)}]^\top$, $\mathbf{x}_\tau \triangleq [x^{(1)}, \dots, x^{(\tau)}]^\top$, $\mathbf{n}_\tau \triangleq [n^{(1)}, \dots, n^{(\tau)}]^\top$, and $\boldsymbol{\Phi}_\tau \triangleq [\boldsymbol{\phi}_1, \dots, \boldsymbol{\phi}_\tau]^\top$. This form is useful in later derivations. When $\tau = T_p$, we replace the subscript τ with p to indicate that the collected signals correspond to all pilot symbols.

C. Conventional Pilot-Based Channel Estimation

Letting $\mathbf{Q}_p \triangleq \sqrt{E_s} \text{diag}(\mathbf{x}_p) \boldsymbol{\Phi}_p$ in (3), the cascaded channel $\mathbf{c}$ can be estimated from the received pilot symbols via

$$\hat{\mathbf{c}}_p = \underset{\tilde{\mathbf{c}} \in \mathbb{C}^N}{\text{argmin}} \|\mathbf{y}_p - \mathbf{Q}_p \tilde{\mathbf{c}}\|^2. \quad (4)$$

For $T_p \geq N$ and a full column rank matrix $\mathbf{Q}_p$, the resulting LS estimate is $\hat{\mathbf{c}}_p = (\mathbf{Q}_p^\dagger \mathbf{Q}_p)^{-1} \mathbf{Q}_p^\dagger \mathbf{y}_p$. Assuming unit-modulus pilot symbols, the NMSE of the channel estimate is defined as [4], [5]:

$$\frac{\mathbb{E}[\|\hat{\mathbf{c}}_p - \mathbf{c}\|^2]}{\mathbb{E}[\|\mathbf{c}\|^2]} = \frac{\sigma^2}{NE_s} \text{tr}[(\boldsymbol{\Phi}_p^\dagger \boldsymbol{\Phi}_p)^{-1}]. \quad (5)$$

Minimizing the trace in (5) requires $\boldsymbol{\Phi}_p^\dagger \boldsymbol{\Phi}_p$ to be a scaled identity matrix, i.e., $\boldsymbol{\Phi}_p^\dagger \boldsymbol{\Phi}_p = T_p \mathbf{I}_N$, as shown in [7]. This condition clearly constrains the selection of the RIS configurations and may be difficult to satisfy under practical phase-shift restrictions.

D. Conventional RIS Configurations

To approach the optimal condition $\Phi_p^\dagger \Phi_p = T_p \mathbf{I}_N$ for arbitrary T_p and N , the works in [4], [5] proposed practical RIS configurations based on nearly orthogonal matrices. The QDFT RIS configuration matrix is obtained by first generating an $T_p \times T_p$ DFT matrix $\mathbf{F}_{T_p}$. The phase of each entry of $\mathbf{F}_{T_p}$ is then quantized to its nearest b -bit phase shift in $\mathcal{F}$, denoted by the operation $\mathcal{Q}_b(\cdot)$, after which the first N columns are selected, i.e.,

$$\Phi_{\text{QDFT}} = [\mathcal{Q}_b(\mathbf{F}_{T_p})]_{:,1:N}. \quad (6)$$

Moreover, the THM RIS configuration is obtained by truncating an $M \times M$ Hadamard matrix $\mathbf{H}_M$, where $M = 2^q$ for some positive integer q and $M \geq \max(T_p, N)$, i.e., the configuration is given by

$$\Phi_{\text{THM}} = [\mathbf{H}_M]_{1:T_p, 1:N}. \quad (7)$$

Note that the THM RIS configuration is limited to binary phase shifts, i.e., $\mathcal{F} = \{0, \pi\}$.

III. DF CHANNEL ESTIMATION WITH GUM RIS CONFIGURATIONS

A. RIS Configuration Derived from GUMs

In addition to the existing constructions, we show that GUMs [10] can also produce nearly orthogonal RIS configurations for even block lengths over suitable finite phase-shift alphabets, as detailed below. Let $(\mathbf{a}, \mathbf{b})$ be a length- L Golay complementary pair. Denote by $\text{circ}(\mathbf{a})$ and $\text{circ}(\mathbf{b})$ the circulant matrices generated by $\mathbf{a}$ and $\mathbf{b}$, respectively. For $M = 2L$, the GUM is constructed as

$$\mathbf{U}_G = \begin{bmatrix} \text{circ}(\mathbf{a}) & \text{circ}(\mathbf{b}) \\ \text{circ}(\mathbf{b})^\dagger & -\text{circ}(\mathbf{a})^\dagger \end{bmatrix}, \quad (8)$$

which satisfies $\mathbf{U}_G^\dagger \mathbf{U}_G = M \mathbf{I}_M$. The corresponding RIS configuration matrix Φ_{GUM} is obtained by quantizing, when needed, the entries of $\mathbf{U}_G$ to $\mathcal{F}$ and selecting the first N columns, i.e.,

$$\Phi_{\text{GUM}} = [\mathcal{Q}_b(\mathbf{U}_G)]_{:,1:N}. \quad (9)$$

The GUM construction enables orthogonal RIS configurations for a wide range of (T, N) with a limited phase alphabet $\mathcal{F}$. Exact orthogonality is guaranteed for any $N \leq T$ if $\mathcal{F}$ contains all phases of the underlying Golay pair. To the best of our knowledge, GUM-based RIS configuration design for channel estimation has not been previously studied.

Example. An $L = 3$ Golay complementary pair is given by $\mathbf{a} = [1, 1, -1]$ and $\mathbf{b} = [1, j, 1]$, which induces the following 6×6 GUM:

$$\mathbf{U}_G = \begin{bmatrix} 1 & 1 & -1 & 1 & j & 1 \\ -1 & 1 & 1 & 1 & 1 & j \\ 1 & -1 & 1 & j & 1 & 1 \\ 1 & 1 & -j & -1 & 1 & -1 \\ -j & 1 & 1 & -1 & -1 & 1 \\ 1 & -j & 1 & 1 & -1 & -1 \end{bmatrix}. \quad (10)$$

B. Decision-Feedback Channel Estimation

DF channel estimation treats detected data symbols as additional pilot symbols, allowing the same LS form as in pilot-based estimation to be applied with an enlarged observation set. This can improve estimation accuracy when the detected symbols are sufficiently reliable. In this paper, we consider two DF schemes: block-wise DF and sequential DF.

The block-wise DF scheme re-estimates the cascaded channel after detecting all data symbols. To mitigate both channel-estimation and data-detection errors, we design the pilot and data RIS configuration matrices so that $\Phi_p^\dagger \Phi_p = T_p \mathbf{I}_N$ and $\Phi_d^\dagger \Phi_d = T_d \mathbf{I}_N$. Assuming perfect data detection, the NMSE of the DF-based channel estimation reduces to

$$\frac{\sigma^2}{NE_s} \text{tr} \left[\left(\Phi_p^\dagger \Phi_p + \Phi_d^\dagger \Phi_d \right)^{-1} \right] = \frac{\sigma^2}{E_s T}. \quad (11)$$

This reduction presents the structural advantage of GUMs, as different choices of Golay sequences can readily support a wider range of T_p , T_d , and N without requiring truncation or even phase quantization.

In contrast, the sequential DF scheme refines the cascaded channel estimates after each detected data symbol. Its performance relies critically on Φ_p since the initial pilot-based channel estimate directly affects the reliability of subsequent data detection. Under the assumption of perfect detection up to time t , it is also desirable to minimize

$$\text{tr} \left[\left(\Phi_p^\dagger \Phi_p + \Phi_{d,1:t}^\dagger \Phi_{d,1:t} \right)^{-1} \right], \quad (12)$$

although maintaining orthogonality of Φ_{T_p+t} for all t is challenging. In this scheme, GUMs are particularly useful for providing accurate initial estimates.

We end this section with two remarks. First, the conventional separate design of Φ_p and Φ_d is mainly motivated by the non-DF setting and the requirement of passive beamforming. When DF is considered and passive beamforming is not the

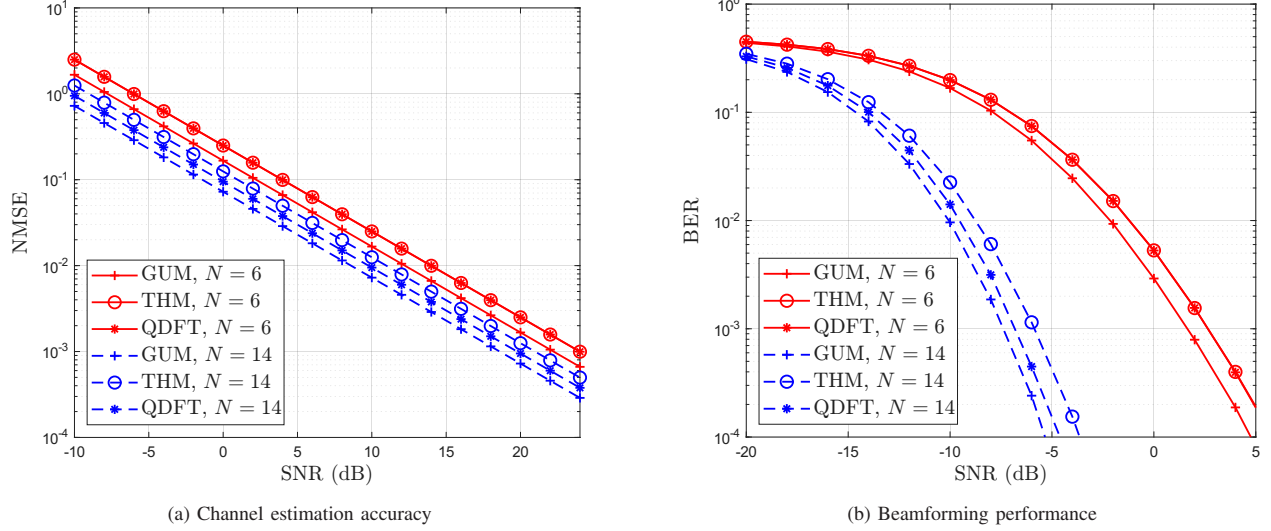


Fig. 3: Performance of conventional pilot-based channel estimation with different RIS phase configurations and $T = 16$.

primary objective, a more natural approach is to design the full RIS configuration matrix Φ_T and sequentially apply its rows over the T symbol intervals. In this case, numerical results show that the THM-based RIS configuration outperforms the GUM-based one because it achieves a higher level of orthogonality. However, since THM is limited to binary phase shifts, its applicability is restricted.

Second, the above DF schemes differ from that in [11], in which only the effective BS-UE channel is estimated under a fixed RIS configuration. Although their approach reduces pilot overhead, it does not recover the cascaded channel information and thus cannot readily support passive beamforming. This distinction is not intended as a performance comparison among DF schemes, but rather to emphasize that DF channel estimation can be implemented in different forms depending on the system objective.

IV. NUMERICAL RESULTS AND DISCUSSION

This section investigates the impact of different RIS configurations on the proposed DF channel estimation and passive beamforming. In all simulations, unless otherwise stated, we set $b = 2$ and employ BPSK modulation. The SNR is defined as $E_b/N_0 = 1/\sigma^2$, where the average energy per information bit E_b is normalized to one and σ^2 is the noise variance. Channel estimation accuracy is measured by the NMSE, while passive beamforming performance is assessed by the detection error rate under RIS phase compensation, where each phase

shift $\theta_i \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ is selected to best compensate for the phase rotation of the corresponding estimated cascaded channel coefficient.

Fig. 3(a) compares conventional pilot-based LS channel estimation under different RIS configurations for $T = 16$ and $N \in \{6, 14\}$. The NMSE decreases with increasing SNR for all configurations. In particular, the GUM configuration achieves the lowest NMSE, indicating that it better preserves the orthogonality required for LS estimation. In contrast, the other configurations exhibit higher NMSEs because matrix truncation and phase quantization degrade this orthogonality. For the downlink passive beamforming based on the estimated cascaded channels, the detection error rate in Fig. 3(b) exhibits a trend consistent with the corresponding NMSE performance. Overall, more accurate channel estimates improve the effectiveness of phase compensation, while a larger number of reflecting elements offers a higher passive beamforming gain.

Fig. 4 demonstrates the impact of $\mathcal{F}$, where the unquantized DFT configuration serves as the lower bound. For $b = 1$, the QDFT configuration Φ_{QDFT} is excluded since it is rank deficient; the GUM configuration outperforms the THM configuration for most values of N , except at $N = 14$. For $b = 2$, the GUM configuration generally achieves an NMSE closer to the lower bound than the QDFT configuration. As b increases, the quantization error of the QDFT configuration decreases, and their performance difference becomes negligible. We note that the performance loss of GUM configurations at

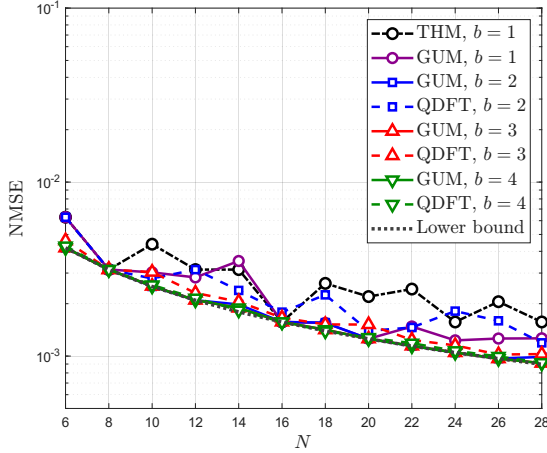


Fig. 4: NMSE comparison of THM, QDFT, and GUM RIS configurations versus N at an SNR of 16 dB for $b \in \{1, 2, 3, 4\}$.

$N \in \{14, 18, 28\}$ is due to phase quantization.

Lastly, for $T = 18$ and $N = 6$, Fig. 5 compares the conventional LS estimator, which uses only pilot symbols, with the block-wise and sequential DF schemes. For all three RIS configuration matrices, both DF schemes reduce the NMSE, confirming the benefit of reusing detected data symbols for channel refinement. The block-wise DF scheme generally achieves the lowest NMSE, since it re-estimates the channel after collecting all detected data symbols and is therefore less affected by error propagation. By contrast, the sequential DF scheme updates the channel after each detected symbol and is more sensitive to erroneous decisions. Under each estimation scheme, the GUM-based RIS configuration consistently provides the best performance, demonstrating its advantage in preserving the orthogonality required for accurate LS-based channel estimation.

V. CONCLUSION AND FUTURE WORK

In this work, we illustrated that GUM-based RIS configurations provide a more flexible near-orthogonal design for purely passive RIS-assisted communications under finite phase-shift constraints, achieving performance comparable to or better than conventional designs. The proposed DF schemes further improved cascaded channel estimation, thereby increasing passive beamforming gains. The numerical results suggest that RIS configuration design and DF-based refinement should be considered jointly. Future research directions include investigating the orthogonality requirements of the sequential DF scheme and extending the framework to slow time-varying channels.

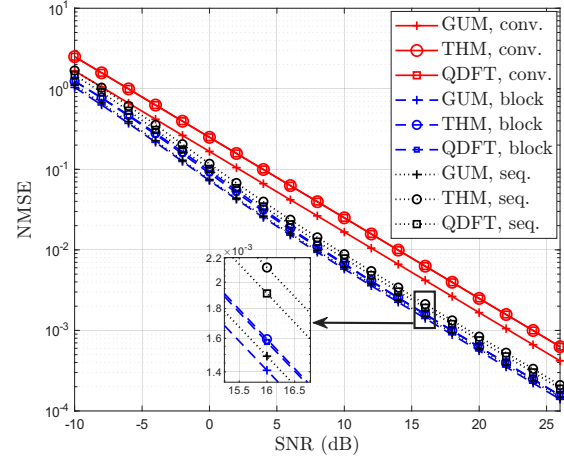


Fig. 5: NMSE comparison of conventional, sequential DF, and block-wise DF channel estimation with different RIS configuration matrices for $T = 18$ and $N = 6$.

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